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THE SEPARATION LOOP AS AN ARCHITECTURAL DISTINCTION BETWEEN CLASSICAL & MODERN CONTROL SYSTEMS

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ABSTRACT

It is argued that straightforward generalization of the architecture of classical single-input single-output (SISO) control systems to multiple-input multiple-output (MIMO) systems introduces several issues which are avoidable in a more appropriate architecture. In fact, active implementation of the architecture implicit in Kalman's concepts of Controllability, Observability, and the Control/Filter Separation Principle appears to avoid such issues as "pole-zero cancellation", "open-loop plant transfer-matrix inversion", "minimum-phase open-loop plants", "right-half-plane transmission zeros," etc. In the ideal case of perfect model identification, the signal-processing poles are identical in the compensator filters considered as testable open-loop components and in the closed-loop controlled system. If maximally rhobust* design procedures are followed, then in reality these poles shift minimally when the feedback loops are closed, provided that feedback of actuator signals is implemented separately from feedback of sensor signals by means* of an active summing element as demonstrated herein. [We use rhobust instead of robust to denote a particularly strong insensitivity to both time-varying and nonlinear perturbations as in the 1956 ρ-synthesis criterion of Bass rather than to strictly LTI perturbations [4].].

Introduction

The classical compensator is taken to be a signal-processing component which links the sensor-suite to the actuator-suite as in Figure 1. It is convenient to reformulate the problem so that the dynamics of both sensors and actuators are included as part of the open-loop plant dynamics, or *process dynamics*, so that the components referring to sensors and actuators in Figure 1 actually pertain only to kinematics. For convenience in order to make the point at hand we consider only continuous-time systems and analog controllers, in which case the compensator's components are to be physically realizable by stable passive networks such as RC circuits or RLC circuits; when these are approximated by digital algorithms updated with sufficiently high frequency, the main effect is a slight degradation of the theoretically-predicted performance which is not considered here.

However, there is one distinction between analog and digital compensators that is relevant to the present discussion. The straightforward generalization of classical SISO practice to the MIMO case would suggest an implementation as in Figure 1 (shown in greater detail in Figure 3). However if the components of the transfer matrix in Figures 1 and 3 are to be realizable by stable passive analog networks, then in general the issues mentioned in the Abstract above (concerning pole-zero cancellation, plant transfer-matrix inversion, right-half-plane transmission zeros, minimum-phase open-loop plants) which limit stability augmentation and robustification by high gain feedback, cannot be avoided.

Moreover, as shown in Figure 2 (and in greater detail in Figures 4 and 5), if one allows the use of an active component to *sum* the outputs of the stable passive components, then (as will be demonstrated) these issues become irrelevant. Furthermore, if the various analog transfer functions of Figures 2, 4, and 5 are approximated by digital filters, then the presence of an active summing element is not an additional consideration as it is in the analog case.

In the sequel we shall use the terms "rhobust" and "rhobustness" (suggested by a Rockwell colleague, Dr. A.P. Andrews) to distinguish between "robust" and "robustness" as it is used in the extensive contemporary multivariable literature [15]-[16] from a frequency-domain viewpoint, to refer to the allowed class of perturbations to be strictly LTI (Linear Time Invariant), and as used by R.W. Bass in 1956 and 1983 wherein *time-varying and nonlinear* perturbations *are allowed* provided that their Lipschitz constant κ of the (*unstructured, norm bounded* [19]) perturbations remains smaller than the Bass ρ-criterion (Bass [1], [2]); furthermore, we restrict this term to the particularly strong insensitivity to perturbations in which $\rho = \rho(A)$ is a true [sub-additive] *margin*: $\rho(A\pm\Delta A) \geq \rho(A)-\|\Delta A\|$, and in which the *overshoot factor* $\gamma = \gamma(A)$ remains *INVARIANT* and in which the *exponential decay rate* $\lambda = \lambda(A)$ is only *MARGINALLY VARIANT* in that it gets replaced by $\lambda\cdot(1 - \kappa/\rho)$ [where, noting that λ and γ are by now means unique, they are so selected as to *maximize* $\rho \equiv \lambda/\gamma$, thus fulfilling a *desideratum* published by Wiberg [18] in 1967 as shown in the companion paper [4]].

Finally, the main attraction of the architecture of Figures 2, 4, and 5 is that it permits use of design procedures (Bass [1]-[4], [10]-[12]) by means of which the engineer may make a rational tradeoff between *rhobustness* of *stability* and *fidelity* in a manner that is uniform throughout the closed-loop system, i.e. the *rhobustness margin* ρ (Bass [1], [2]) which applies *equally* to every point at every loop. In an earlier paper on *rhobust tuning of Kalman filters* (Bass [3]) it was shown how to implement a one-parameter search which displays this tradeoff as a function of a suitable design parameter, or in terms of the definition (Bass [2]) of the *bandwidth* $\omega_{BW} \equiv (\|A\|/\|A^{-1}\|)^{1/2}$ of a stable n-dimensional system whose dynamical coefficient matrix is a Hurwitz matrix A. By making this tradeoff first for the ideal system (assuming that all n state variables are measured), one chooses the desired constellation of the n closed-loop poles corresponding to the open-loop process poles, which partially determines the zeros of the transfer functions of the compensator components shown in Figures 2, 4 and 5 while leaving the n signal-processing poles completely undetermined. A second application of this rhobust-tuning procedure, either to the selection of the filter poles or else (more conservatively) to the entire closed loop determines the preferred positions of the n signal-processing poles. But this two-stage fidelity/stability rhobustness tuning procedure is not available unless the architecture of Figure 2, 4, and 5 rather than that of Figures 1 and 3 is assumed.

Problem Definition

It will be assumed that the reader has some familiarity with state-space methods (cf., e.g. Kailath [5]; Balakrishnan [6], [7]; Friedland [8]; Stengel [9]; Wonham [14]; Hung & MacFarlane [15]; Maciejowski [16]).

Consider the evolution in time of an autonomous LTI system with n *state variables* x_i, k sensed *outputs* y_j, and m control *inputs* u_k. Then the evolution in time t of the system is governed by

$$\dot{x} = Fx+Gu+Q^{1/2}v(t),\ x(0) = x^0,\ (\dot{} = d/dt,\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m) \quad (1)$$

$$y = Hx + R^{1/2}w(t),\ (y \in \mathbb{R}^k), \quad (2)$$

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where the *process noise* (or *plant disturbance*) $v(t)$ and the *sensor noise* $w(t)$ are zero mean stationary Gaussian random processes of identity-matrix intensity covariances I_n and I_m, respectively, and where (x,u,y) are the *state*, *input*, and *output vectors*, respectively. The effective disturbance and noise intensity covariances Q and R are non-negative definite symmetric matrices which satisfy

$$Q = Q' \geq 0, \quad R = R' > 0, \tag{3}$$

where $'$ denotes matrix transposition, and where R is required to be strictly positive definite. If there is any cross correlation between the process disturbance v and the sensor noise u, given by a non-vanishing cross-intensity matrix S, then it is usual to ensure the existence of a minimal-variance Kalman-Bucy state-estimator by requiring additionally that Q not only be positive-definite, but that S be sufficiently small that

$$Q - S\cdot R^{-1}\cdot S' > 0 . \tag{3bis}$$

The **problem** is to construct a feedback controller u as a linear functional of the past observations of y, namely $\{y(\tau) \mid 0 \leq \tau \leq t\}$, so as to make the equilibrium solution $x \equiv 0$ globally exponentially asymptotically stable.

Problem Solution

Now postulate a *linear* (state-estimation) *filter* which produces an (asymptotic, approximate) estimate of x, say $\hat{x}$, by a signal-processing & algorithmic architecture of the *linear* form

$$\dot{\hat{x}} = F\hat{x} + Gu + L(y - H\hat{x}), \quad \hat{x}(0) = 0, \tag{4}$$

where the $n \times k$ matrix L is an unknown *filter gain matrix*, which operates (so as to produce "negative" error feedback) on the *innovation* $(y - H\hat{x})$, which is the *residual error* between the sensed output y and its estimate $\hat{y} = H\hat{x}$. We shall suppose that L can be chosen to render

$$\tilde{F} \equiv F - LH \tag{5a}$$

a *Hurwitz* matrix, i.e. one whose *poles* or *characteristic roots* all have negative real parts, and whose characteristic polynomial

$$\tilde{\Delta}(s) \equiv det(sI_n - \tilde{F}) \equiv s^n + \tilde{\alpha}_{n-1}s^{n-1} + \cdots + \tilde{\alpha}_1 s + \tilde{\alpha}_0 \tag{6}$$

has as roots the *filter poles* or "electronic poles". This can always be done if the pair (F,H) satisfies Kalman's criterion of *observability*, in which case (as first proved algebraically in the case $m = 1$ by Bass [10, 1961]) the roots of (6), or equivalently the coefficients $\{\alpha_i\}$ are arbitrarily prespecifiable, and at least one corresponding L can be found, e.g. by the Bass-Gura formula [5, p. 198], [8, p. 230]. As is also well known, if one seeks a *minimal variance* estimate and has confidence in the assumed values of Q, R, and S then L can be chosen to be the optimal filter gain according to the *Kalman-Bucy Filter* theory providing L as the solution of a quadratic matrix equation called the ARE (Algebraic Ricatti Equation) and defined solely in terms of (F,H,Q,R,S).

Now the filter can be expressed as a stable subsystem

$$\dot{\hat{x}} = \tilde{F}\hat{x} + Gu + Ly, \quad \hat{x}(0) = 0, \tag{7a}$$

$$\hat{x}(s) = \{(sI_n - \tilde{F})^{-1}G\}u(s) + \{(sI_n - \tilde{F})^{-1}L\}y(s), \tag{7b}$$

where s denotes complex frequency and where $(sI_n - \tilde{F})^{-1} \equiv \tilde{\Gamma}(s)/\tilde{\Delta}(s)$ is the *resolvent matrix* of $\tilde{F}$; here $\tilde{\Gamma}(s) \equiv \sum_{i=1}^{n} s^{i-1}S_i$ is an $n \times n$ matrix each of whose elements is a polynomial in s of degree at most $(n - 1)$.

Suppose that we desire to use *linear state feedback* to control (1), and suppose that the $m \times n$ matrix K is *ideal* state-feedback *gain matrix*, in the sense that if $k = n$, $H = I_n$, $v(t) \equiv 0$, $w(t) \equiv 0$, then the *deterministic* control law

$$u = -Kx \tag{8}$$

would provide "negative" feedback which would stabilize (1), in that it would then become an *ideal closed-loop system*

$$\dot{x} = \hat{F}x, \quad x(0) = x^0, \quad (\dot{} = d/dt, \; x \in \mathbb{R}^n) \tag{9}$$

$$\hat{F} \equiv F - GK \tag{5b}$$

where now $\hat{F}$ is a Hurwitz matrix of characteristic polynomial

$$\hat{\Delta}(s) \equiv det(sI_n - \hat{F}) \equiv s^n + \hat{\alpha}_{n-1}s^{n-1} + \cdots + \hat{\alpha}_1 s + \hat{\alpha}_0, \tag{10}$$

whose n roots are the *ideal plant* [*process*] *poles* or ideal "electromechanical" poles. Such a K can always be found if the pair (F,G) satisfies the condition of *controllability* [Kalman, 1960], in which case by what Kalman has called *The Fundamental Theorem of Control Theory* (Bass [10, 1961]) the ideal plant poles are arbitrarily prespecifiable; as is well known ([5]-[9]), if one chooses K to minimize the integral over all future time of a quadratic form in the $(n + m)$-vector $(x',u')'$, then K can be taken to be the Optimal Gain [Kalman, 1960] corresponding to the *Kalman Regulator Law*, and such a *Kalman Gain Matrix K* can be found by solving an ARE defined solely by F, G and the matrices defining the quadratic form.

Now return to the *stochastic* case and consider the so-called *certainty equivalence* control law

$$u = -K\hat{x}, \tag{11}$$

which would coincide with the ideal control law if $\hat{x}$ were a perfect estimate of x, i.e. if the *error state*

$$\tilde{x} = x - \hat{x} \tag{12}$$

were identically zero. If one substitutes (2) in (4) and then subtracts (4) from (1), one obtains, after substituting (11) in (1) and using (12), the *total closed-loop* system in the form

$$\dot{x} = \hat{F}x + GK\tilde{x} + Q^{1/2}v(t), \; x(0) = x^0, \quad (\dot{} = d/dt, \; x \in \mathbb{R}^n), \tag{13a}$$

$$\dot{\tilde{x}} = \tilde{F}\tilde{x} + Q^{1/2}v(t) - LR^{1/2}w(t), \; \tilde{x}(0) = x^0, \; (\tilde{x} \in \mathbb{R}^n). \tag{13b}$$

Notice that (13a,b) can be expressed more concisely as

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{Q}^{1/2}\mathbf{v}(t), \; \mathbf{x}(0) = \mathbf{x}^0, \; (\dot{} = d/dt, \mathbf{x} \in \mathbb{R}^{2n}, \mathbf{v} \in \mathbb{R}^{n+m}), \tag{14a}$$

$$\mathbf{x} \equiv \begin{pmatrix} x \\ \tilde{x} \end{pmatrix}, \quad \mathbf{v} \equiv \begin{pmatrix} v \\ w \end{pmatrix}, \tag{14b}$$

$$\mathbf{F} = \begin{pmatrix} \hat{F}, & GK \\ 0, & \tilde{F} \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} Q^{1/2}, & 0 \\ Q^{1/2}, & -LR^{1/2} \end{pmatrix}, \tag{14c}$$

where now one has the *Algebraic Separation Theorem* (Bass [11, 1964])

$$det(s\mathbf{I}_{2n} - \mathbf{F}) \equiv \hat{\Delta}(s)\cdot\tilde{\Delta}(s), \tag{15}$$

which is more primitive than (hence more fundamental than) what is now called the *LQG Separation Principle* of stochastic optimal control theory (as first used by Kalman, Englar and Bucy in 1962). The Algebraic Separation Theorem shows that *the controlled system's closed-loop poles consist of the union of its ideal electromechanical poles and its* [filter-added] *electronic* [or signal-processing] *poles*. Thus the effect of the filter (7) and the ideal control law (8) used in the certainty-equivalence form (11) is to shift the *open loop* system's poles, i.e. the roots of

$$\Delta(s) \equiv det(sI_n - F) \equiv s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0, \tag{16}$$

to their ideal electromechanical pole positions *at the cost of adding n new poles*, namely the electronic or signal-processing poles.

Consequences

Another way of displaying the *architecture* of the total closed-loop system is as

$$\dot{x} = Fx + Gu + Q^{1/2}v(t), \; x(0) = x^0, \; y = Hx + R^{1/2}w(t), \tag{17a}$$

$$u(s) = \Phi_{sep}(s)\cdot u(s) + \Phi(s)\cdot y(s), \tag{17b}$$

$$\Phi_{sep}(s) \equiv -K(sI_n - \tilde{F})^{-1}G \equiv -\{K\tilde{\Gamma}(s)G\}/\tilde{\Delta}(s), \tag{17c}$$

$$\Phi(s) \equiv -K(sI_n - \tilde{F})^{-1}L \equiv -\{K\tilde{\Gamma}(s)L\}/\tilde{\Delta}(s), \tag{17d}$$

where the $m \times m$ matrix $\{K\tilde{\Gamma}(s)G\}$ has only polynomial elements of degree at most $(n - 1)$, as does the $m \times k$ polynomial-element matrix $\{K\tilde{\Gamma}(s)L\}$.

In the implementation (17), the electronic signal-processing poles, or roots of $\tilde{\Delta}(s)$, are the identical poles of two separate components, the *separation filter* and the *feedback filter*; these components are physically realizable [in an analog implementation] as *stable, passive* RC or RLC networks -- the only *active* element needed is a summer to combine the outputs of the two components. (See Figures 2, 4, and 5.) Thus each component can be laboratory bench-tested to confirm that it has the desired electronic poles.

Amazingly, when the two components are included in the *closed-loop* implementation (17a,b), the *electronic poles do not shift!*

This point seems to have been forgotten by many otherwise well-informed practitioners and is often denied or doubted when they are reminded of it.

Presumably the following historical explanation accounts for this skepticism.

On first sight of (17a,b,c,d) it is tempting to *rearrange* the *controller* subsystem into the classical controller form

$$u(s) \;=\; \Psi(s)\cdot y(s), \tag{18}$$

where the $m \times k$ *controller transfer matrix*

$$\Psi(s) \;\equiv\; -\{\tilde{\Delta}(s)I_m + [K\tilde{\Gamma}(s)G]\}^{-1}\cdot\{K\tilde{\Gamma}(s)L\} \tag{19a}$$

has as its poles the roots of

$$det\{\tilde{\Delta}(s)I_m + [K\tilde{\Gamma}(s)G]\}, \tag{19b}$$

and where *UNEXPECTEDLY* the *electronic poles* [roots of $\tilde{\Delta}(s)$] have *canceled perfectly* out of (19a) and have become *irrelevant* (in the sense that the poles themselves are not *directly* relevant, though obviously *indirectly* relevant through their symmetric functions which provide the coefficients of the polynomial Δ in (19b))! To see this, note that

$$u(s) = \{I_m - \Phi_{sep}(s)\}^{-1}\cdot\Phi(s)\cdot y(s) \equiv$$

$$\equiv [I_m + \{K\tilde{\Gamma}(s)G/\tilde{\Delta}(s)]^{-1}\cdot[-\{K\tilde{\Gamma}(s)L\}/\tilde{\Delta}(s)]\cdot y(s) \equiv$$

$$\equiv -\{\tilde{\Delta}(s)/\tilde{\Delta}(s)\}\cdot[\tilde{\Delta}(s)I_m + \{K\tilde{\Gamma}(s)G/\tilde{\Delta}(s)]^{-1}\cdot\{K\tilde{\Gamma}(s)L\}\cdot y(s) \equiv$$

$$\equiv \Psi(s)\cdot y(s) \tag{20a}$$

as claimed. Thus, having shown *explicitly* that the filter poles cancel out of the usual formulation of (18)-(19), we shall now demonstrate that (18)-(19) is in fact identical to the usual form, if in the course of the demonstration we happen to be sufficiently inspired or lucky as to *insert* the non-obvious factor

$$1 \;\equiv\; \{[\tilde{\Delta}(s)]^{-1}\}^{-1}/\tilde{\Delta}(s) \tag{19c}$$

at a cleverly chosen step; but first, we need a preliminary matrix-identity lemma. If K is an arbitrary $m \times n$ matrix, and $-\delta \neq 0$ is an arbitrary scalar which is not a root of either $det(\delta I_m + KA) = 0$ or $det(\delta I_n + AK) = 0$, where A is an arbitrary $n \times n$ matrix, then in general it is trivial to verify the identity $(\delta I_m + KA)K \equiv K(\delta I_n + AK)$ whence one has the less obvious matrix-inversion/dimension-reduction identity

$$K(\delta I_n + KA)^{-1} \;\equiv\; (\delta I_m + AK)^{-1}K\ . \tag{20b}$$

Now take $\delta = \tilde{\Delta}$ and $A = \tilde{\Gamma}G$ in (20b) and rearrange (19a); get

$$\Psi(s) \;\equiv\; -\{\tilde{\Delta}(s)I_m + [K\tilde{\Gamma}(s)G]\}^{-1}\cdot\{K\tilde{\Gamma}(s)L\} \;\equiv$$

$$\equiv\; -1\cdot K\{\tilde{\Delta}(s)I_n + [\tilde{\Gamma}(s)GK]\}^{-1}\cdot\{\tilde{\Gamma}(s)L\} \;\equiv$$

$$\equiv\; -[\{[\tilde{\Delta}(s)]^{-1}\}^{-1}/\tilde{\Delta}(s)]\cdot K\{\tilde{\Delta}(s)I_n + [\tilde{\Gamma}(s)GK]\}^{-1}\cdot\{\tilde{\Gamma}(s)L\} \;\equiv$$

$$\equiv\; -K\{I_n + [\{\tilde{\Gamma}(s)/\tilde{\Delta}(s)\}GK]\}^{-1}\cdot\{\tilde{\Gamma}(s)/\tilde{\Delta}(s)\}L \;\equiv$$

$$\equiv -K\{I_n + [(sI_n - \tilde{F})^{-1}GK]\}^{-1}\cdot(sI_n - \tilde{F})^{-1}L \equiv$$

$$\equiv -K\{(sI_n - \tilde{F})[I_n + (sI_n - \tilde{F})^{-1}GK]\}^{-1}L \equiv$$

$$\equiv\; -K(sI_n - \tilde{F} + GK)^{-1}L \;\equiv\; -K(sI_n - [F - LH - GK])^{-1}L \;\equiv$$

$$\equiv\; -K(sI_n - \tilde{\tilde{F}})^{-1}L\ , \tag{19d}$$

where

$$\tilde{\tilde{F}} \;\equiv\; F - LH - GK \;\equiv\; \tilde{F} - GK \;\equiv\; \hat{F} - LH \tag{19e}$$

is the dynamical coefficient matrix of the controller as *usually* specified ([8]-[9]).

But in general, Ψ will have poles in the RHP (Right Half Plane) of the complex- frequency plane!

In his excellent text on control system design [8], B. Friedland points out quite explicitly several times (e.g. p. 295, p. 306) that when any of the open-loop plant transmission zeros are in the RHP (right half-plane), then "the compensator could turn out to be unstable!", and this can lead to "a remedy worse than the problem it cures"! (But implementation of an algebraically *equivalent* compensator via the Separation Loop Architecture advocated here renders this problem irrelevant.)

Similarly in his fine text on stochastic optimal control [9], R.F. Stengel points out that "compensator instability is possible" in his equation (6.6-6) (p. 604) and mentions negative practical consequences (p. 605). (But these consequences can be avoided by the procedures illustrated here.) Thus it is well-known that the controller (18)-(19) cannot always be physically realized by stable, passive RC/RLC networks, but in general requires active elements *throughout*, not just at a *single* summing junction as in the presently advocated architecture (17).

The only way to avoid these difficulties is to give up (18) *and return to* (17)*!*

Conclusions

This is a deep structural difference between classical and modern control theory. Starting with classical concepts, one would automatically postulate the architecture (18) of Figures 1 and 3; the electronic poles *would never be discovered* [unless, "cheating" by using time-domain-based hindsight, one arbitrarily postulated the *non- obvious factorization* (19a) -- but it seems fair to call this "cheating" because the discovery of putting the factor 1 into (20) and then replacing it by the identity $1 \equiv \equiv \{\tilde{\Delta}(s)/\tilde{\Delta}(s)\}$ as in (19c) without prior knowledge of the importance of $\tilde{\Delta}(s)$ seems infinitely unlikely!].

To see this, try starting out with Figures 1 and 3, and (18), and then in a *natural way* deriving the factorization (19a)! There is no obvious reason to postulate such peculiar structure for $\Psi(s)$. Furthermore, if the properties of pole-invariance between open-loop compensator-components and their corresponding closed-loop positions, etc., summarized above, seem valuable, then from that point of view the postulation of some other form for Ψ would constitute an *irrecoverable* error.

The preceding point is so trivial that many persons familiar with state-variable control theory have forgotten it or cannot be troubled to think carefully about it.

The "modern architecture" advocated herein *completely avoids* the "pole-zero cancellation" issues which arise when one *uses* the classical architecture (18). In particular, whether or not the open-loop plant is "minimum phase" or has "RHP transmission zeros" is irrelevant (whenever, as shown below, the gain matrices K and L are chosen by ρ-synthesis [4] rather than by any of the methods which may produce extremely high gains, such as LQG/LTR, $\mathcal{H}_\infty$, or μ-synthesis).

Furthermore, when one "closes the loop" by implementing (17b,c,d), the poles of the open-loop components (17c) and (17d) do *not* shift, but remain *invariant!*

However, if one had used the classical structure (18), then when one closes the loop the poles of $\Psi(s)$ in (18) would *migrate* to *unpredictable* and possibly unacceptable [even unstable!] locations.

Admittedly, the presently advocated marvelous advantages of the *separation loop* (17c) are not *EXACTLY* maintained when the actual open-loop plant dynamical coefficient matrix F deviates from that assumed in designing the filter (17c,d), i.e. from that assumed in (5). But in this case if (K,L) are so chosen as to *maximally rhobustify* the closed-loop system (as showed by Bass [2]-[4] using the *rhobustness margin* ρ of Bass [1]) then the shift of the closed-loop poles (6) from their ideal values is very small and the advantages of the separation loop remain valid.

This is not the place to detail the rhobustness theory ([1]-[4], [10]-[12]) but a brief indication is in order. The ρ-synthesis theory permits the choice of K and L each to be optimized by an easily-mechanized one-parameter search which maximizes the rhobustness ρ of the corresponding ideal closed loop plant and filter, respectively. Specifically, there are many values of (γ,λ) and (γ_f,λ_f) which satisfy the following inequalities, but there is only one set which *maximizes* each of their *ratios* $\rho = \lambda/\gamma$ and $\rho_f = \lambda_f/\gamma_f$, respectively. Assume that $t \geq 0$, and consider

$$\|exp(\hat{F}t)\| \equiv \|exp([F - GK_{opt}]t)\| \leq \gamma\cdot e^{-\lambda t}, \quad \rho = \lambda/\gamma, \tag{21a}$$

$$\|exp(\tilde{F}t)\| \equiv \|exp([F-L_{opt}H]t)\| \leq \gamma_f\cdot e^{-\lambda_f t}, \; \rho_f = \lambda_f/\gamma_f, \tag{21b}$$

where K_{opt} and L_{opt} have been chosen to maximize ρ and ρ_f. The two systems defined by (13a,b) are now *rhobust* in the sense that if we replace them by the deterministic averaged equations and for the moment ignore their coupling (i.e. delete the $GK\tilde{x}$ term) but include arbitrary Lipschitzian nonlinearities and/or time-varying deviations as well as bounded time-varying external perturbations, i.e.

$$\dot{x} = \hat{F}x + \hat{f}(t,x)+\hat{g}(t), \; x(0) = x^0, \; (\dot{\ } = d/dt, \; x \in \mathbb{R}^n), \tag{21c}$$

$$\dot{\tilde{x}} = \tilde{F}\tilde{x} + \tilde{f}(t,\tilde{x}) + \tilde{g}(t), \quad \tilde{x}(0) = \tilde{x}^0, \; (\tilde{x} \in \mathbb{R}^n), \tag{21d}$$

$$\{\|\hat{f}(t,x)\|/\|x\|\} \leq \kappa < \rho , \quad \{\|\tilde{f}(t,x)\|/\|x\|\} \leq \kappa_f < \rho_f, \tag{21e}$$

$$\|\hat{g}(t)\| \leq \delta, \qquad \|\tilde{g}(t)\| \leq \delta_f , \tag{21f}$$

then the overshoot factors γ and γ_f remain *invariant* (!) while the exponential decay envelopes have the decay factors λ and λ_f increased only by the factor$(1 - \kappa/\rho)$ and the factor $(1 - \kappa_f/\rho_f)$, respectively, in that now, as proved by Bass in 1956 [1], for all $t \geq 0$,

$$x(t)\| \leq \gamma\|x^0\|e^{-\lambda' t}+[\delta/(\rho-\kappa)](1-e^{-\lambda' t}), \; \lambda' \equiv \lambda\cdot(1-\kappa/\rho), \tag{21g}$$

$$\|\tilde{x}(t)\|\leq\gamma\|\tilde{x}^0\|e^{-\lambda'' t}+[\delta_f/(\rho_f-\kappa_f)](1-e^{-\lambda'' t}),\lambda''\equiv\lambda_f(1-\kappa_f/\rho_f). \tag{21h}$$

Now by modifying the sensor suite to improve its satisfaction of the observability criterion, if necessary, or by simply adjusting L minutely so that λ_f is shifted an arbitrarily small amount from its rho-optimal value, we can assume that

$$\lambda_f \neq \lambda . \tag{22}$$

(Some designers may be tempted to simply choose L so that $\lambda_f \gg \lambda$, but this is a grave mistake, because as was noted explicitly by Wiberg [18] in 1967, and emphasized explicitly by Bass in 1983 [2], and has been recently discussed in greater generality by Sussman & Kokotovic [13], the existence of the "peaking phenomenon" may ruin the rhobustness of the total closed loop system if one attempts to make λ_f arbitrarily large.) Next, solving (13) in the deterministic case,

$$x(t) = exp(\hat{F}t)\cdot x^0+(\int_0^t exp(\hat{F}[t - \tau])\cdot GK\cdot exp(\tilde{F}\tau)\cdot d\tau)\cdot x^0, \tag{23}$$

whence application of (21) and simple manipulations yield

$$\{\|x(t)-exp(\hat{F}t)x^0\|/\|x^0\|\}\leq\gamma\gamma_f\|GK\|exp(-\lambda t)(\int_0^t exp(-[\lambda_f-\lambda]\tau)d\tau)\equiv$$

$$\equiv \gamma\cdot\gamma_f\cdot\{(e^{-\lambda_f t} - e^{-\lambda t})/(\lambda - \lambda_f)\}\cdot\|GK\| \equiv$$

$$\equiv \gamma\cdot\gamma_f\cdot\{(e^{-\lambda t} - e^{-\lambda_f t})/(\lambda_f - \lambda)\}\cdot\|GK\| , \tag{24}$$

where in the necessarily *positive* term on the right-hand side of (24) it *does not matter* whether $\lambda > \lambda_f$ or conversely! (This consequence of the present *true* LQG architecture is essential to the success of ρ-synthesis [4].) Hence, in the presence of the ***presently advocated architecture*** the ***effect of the presence of the filter ALWAYS goes exponentially to zero as time increases!*** Moreover, integrating (24), we find that

$$\int_0^{+\infty}\{\|x(\tau) - exp(\hat{F}\tau)\cdot x^0\|/\|x^0\|\}\cdot d\tau \leq \{[\|GK\|/\rho]/\rho_f\}, \tag{25a}$$

which is a *constant* independent of x^0 ! Users of ***high-gain*** synthesis procedures (such as LQG/LTR, $\mathcal{H}_\infty$, or μ-synthesis) may object that this constant may not be acceptably small, and, therefore, the RHP zeros can be important after all. That is true at least in the case of μ-synthesis, where it is demonstrated in [4] on a realistic example that maximizing robustness in the conventional sense increases the denominator of $\{[\|GK\|/\rho]/\rho_f\}$ a mere 50% at the *price* of increasing the numerator by almost 10^7! In fact, in ρ-synthesis, one first *minimizes* not $[1/\rho]$ but $[\|GK\|/\rho]$! Then one minimizes $(1/\rho_f)$ *separately!* (This can only be done in the presently advocated architecture.) An even sharper result than (25a) is

$$\int_0^{+\infty}\{\|x(\tau) - exp(\hat{F}\tau)\cdot x^0\|/\|x^0\|\}\cdot d\tau \leq \{1/\rho(\mathbf{F})\} , \tag{25b}$$

where $\mathbf{F}$ is the $2n\times2n$ matrix defined in (14c). Also, as shown in [4], the best way of choosing L is not simply to minimize $(1/\rho_f)$, but to minimize the RHS of (25b).

The result (25) constitutes an ***EXPLICIT PROOF*** of the near-irrelevance of "transmission zeros" or "open-loop plant zeros" for control systems designed by the present architecture and ρ-synthesis software [12].

Conceivably artificial problems can be invented in which the contrary holds, but the quantitative relationships (24)-(25) can be used to prevent these effects from having any primary significance in a system designed by ρ-synthesis.

For an illustrative worked example, see Figures 6 and 7, wherein the open-loop plant is both unstable and non-minimum phase, and where the gain matrices K and L have been so chosen as to render the algebra easy rather than to achieve optimal performance: in this example, ***instability*** of the usual controller implementation is manifestly unavoidable! For an example where K, L are found by ρ-synthesis, see [4].

Another way to see that the effects of the open-loop zeros are actually only minor, secondary issues when rhobustified modern time-domain control system design theory [12] is employed is to note that, by (13), the effects of (F,H) *alone* define the *second-order* error term $\tilde{x}$, which not only goes exponentially to zero with time but whose relative effect on x, by (24), also goes exponentially to zero with time. But all three matrices (F,G,H) necessary to even *define* the transmission zeros of the open-loop transfer function

$$\Phi_0(s) \equiv H(sI_n - F)^{-1}G \tag{26}$$

only affect jointly the *filter-effect* term $\{GK\tilde{x}(t)\}$ in (13a), which converges to zero exponentially to zero. This filter-effect term may be undesirably large in ***high-gain*** systems with a large $\|K\|$, but maximally *rhobust* systems also have *minimal cost of control*, i.e. minimal rms values of GKx, and are therefore *never* "high gain" systems when compared with gains chosen by other methodologies. Hence if the process observability, which can be defined in terms *only* of the plant-sensor pair (F,H) [and to which the open-loop zeros that depend on (F,G) and the transmission zeros which depend upon (F,G,H) are therefore *irrelevant*, because definable with no reference to G whatsoever], is sufficiently good that the filter rhobustness ρ_f is sufficiently good, in the sense that the RHS of (25a) is not much larger than the RHS of (25b), then the filter-effect term $\{GK\tilde{x}(t)\}$ is rather negligible. Furthermore (24) shows that, contrary to high-gain folklore, what is needed is *NOT* that the filter should be "faster" ($\lambda_f > \lambda$) than the certainty-equivalent ideal closed-loop plant, but that the filter's *effective* time constant ($1/\lambda_f$) should be sufficiently *different* from the ideal closed-loop plant's *effective* time constant (where by *effective* is meant that exaggeratedly large values of $\gamma \cdot \gamma_f$ are avoided by choosing (K,L) not to minimize $([1/\lambda],[1/\lambda_f])$ but instead to minimize $([\gamma/\lambda],[\gamma_f/\lambda_f]) \equiv ([1/\rho],[1/\rho_f])$, which is only possible in the present architecture).

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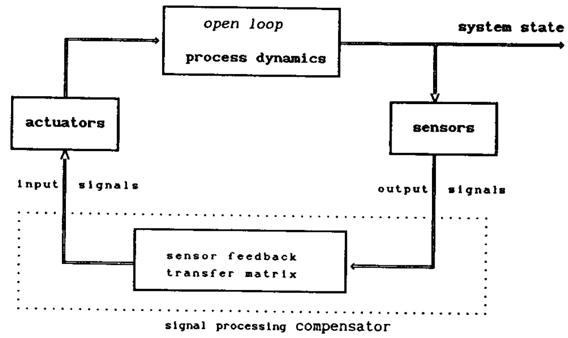
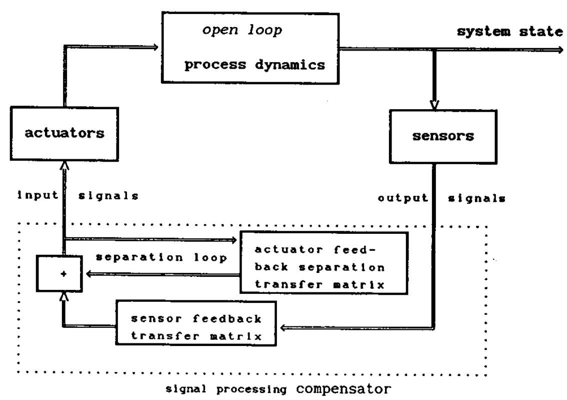


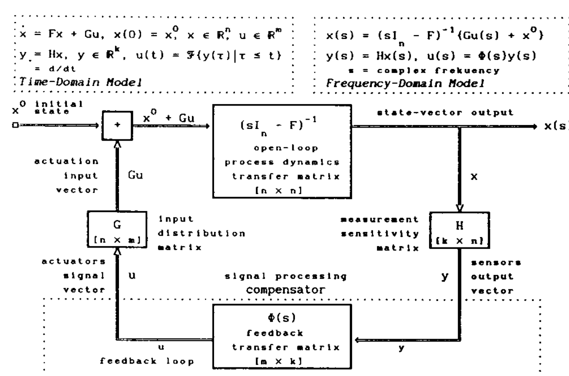
CLASSICAL DESIGN

Figure 1



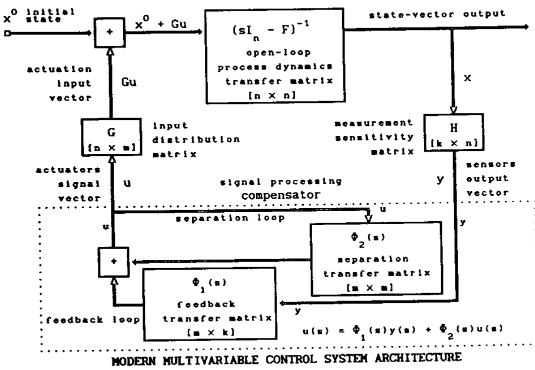
MODERN DESIGN

Figure 2



CLASSICAL MULTIVARIABLE CONTROL SYSTEM ARCHITECTURE

Figure 3



MODERN MULTIVARIABLE CONTROL SYSTEM ARCHITECTURE

Figure 4

ideal control law: $u = -Kx$, K such that $\hat{F} = F - GK$ is Hurwitz, i.e. its resolvent $(sI_n - \hat{F})^{-1}$ has only LHP poles

certainty-equivalence control law: $u = -K\hat{x}$, $\hat{x}(s) = (sI_n - \tilde{F})^{-1}(Gu(s) + Ly(s))$

L such that $\tilde{F} = F - LH$ is Hurwitz

$\tilde{x} = x - \hat{x}$, $\tilde{x}(s) = (sI_n - \tilde{F})^{-1}x^0$, $u(s) = -Kx(s) + K\tilde{x}(s)$,

$x(s) = (sI_n - \hat{F})^{-1}(x^0 + GK\tilde{\tilde{x}}(s))$

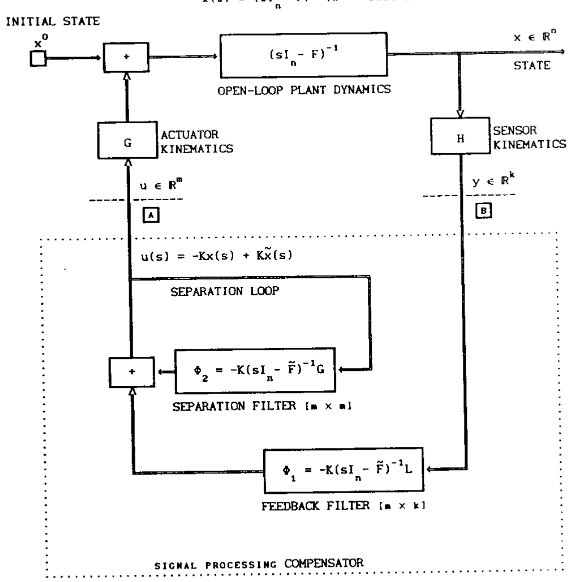
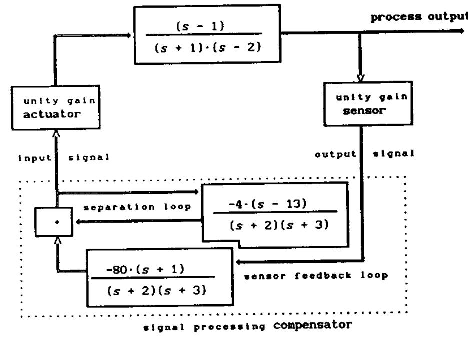


Figure 5

NON-MINIMUM PHASE UNSTABLE PROCESS



ASSIGNED FILTER POLES: $\lambda_{f1} = -2$, $\lambda_{f2} = -3$

ASSIGNED CLOSED-LOOP PLANT POLES: $\lambda_{c11} = -1$, $\lambda_{c12} = -2$

by Bass-Gura pole-placement formula

MODERN DESIGN

Figure 6

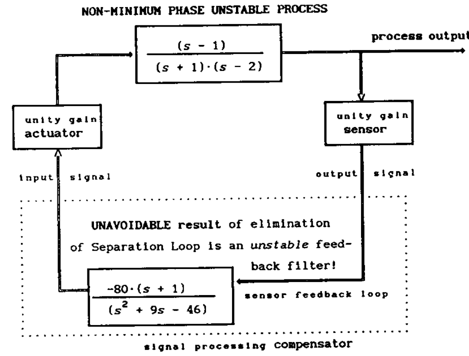
To verify: $y = \Phi_{ol}\cdot u$, $u = \Phi_{sep}\cdot u + \Phi_f\cdot y$, whence $(1 - \Phi_{sep} - \Phi_f\cdot\Phi_{ol})\cdot u \equiv 0$.

Define $\Delta_4 = (s + 1)(s + 2)(s + 3)(s - 2)\cdot\{1 - \Phi_{sep} - \Phi_f\cdot\Phi_{ol}\} \equiv$

$\equiv (s + 1)(s + 2)(s + 3)(s - 2) + 4(s - 13)(s - 2)(s + 1) + 80(s + 1)(s - 1) \equiv$

$\equiv (s + 1)(s + 2)(s + 3)(s + 2) \equiv$ (closed-loop poles)$\cdot$(filter poles), as claimed.

NON-MINIMUM PHASE UNSTABLE PROCESS



ASSIGNED FILTER POLES: $\lambda_{f1} = -2$, $\lambda_{f2} = -3$

ASSIGNED CLOSED-LOOP PLANT POLES: $\lambda_{c11} = -1$, $\lambda_{c12} = -2$

by Bass-Gura pole-placement formula

NEO-CLASSICAL DESIGN

Figure 7

To verify: $u(s) = \Psi(s)\cdot y(s)$, where $\Psi = -K(sI_2 - \tilde{\tilde{F}})^{-1}L$ and where here

$\tilde{\tilde{F}} = F - GK - LH$, $F = \begin{pmatrix} 2 , & 1 \\ 0 , & -1 \end{pmatrix}$, $G = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $H = (1 , 1)$, $K = (12 , 4)$

and $L = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$. Hence, defining $\tilde{\tilde{\Delta}}(s) \equiv s^2 + 9s - 46$, one has, as claimed, that

$\tilde{\tilde{\Delta}}(s)\cdot\Psi(s) = -(12 , 4)\cdot\begin{pmatrix} s + 4 , & -6 \\ -11 , & s + 5 \end{pmatrix}\cdot\begin{pmatrix} 7 \\ -1 \end{pmatrix} \equiv -80\cdot(s + 1)$.