Generalization of the Bass Theory of Laser-Spark Fireballs

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Abstract. Laser spark experiments have been well studied. A novel device, the Bass self-insulated Plasmasphere concept, US Patent 4,448,743, is intended for analysis of high temperature plasmas. The focus of this paper is to present a generalization of the Bass theory and to produce results for both Hydrogen and Helium. This effort is aimed at simulating a fireball that expands to a desired radius whereby the temperature and density are predicted by the proposed theory. The final state momentarily permits a predictable state since volume expansion stops. Therefore, the Plasmasphere concept is appropriate for continued laser spark experimental studies.

Keywords: Laser-spark, fireballs, plasma
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INTRODUCTION

Laser-spark experiments may be designed to generate a laser sustained plasma (LSP). The concept is to focus a remote beam into an absorption chamber, where it is absorbed by the propellant gas [1]. If conditions result in more energy lost than absorbed by the LSP, the LSP becomes unstable and extinguishes, which is commonly referred to as blowout [2].

Applications for LSP have not been widespread although a fair amount of attention remains on laser rocket propulsion [3, 1]. Recent experimental efforts by Toyoda [4] and Mertogul and Krier [2] increase the effectiveness of LSPs for use in propulsion systems. Hot fusion is one application of interest in the current study. One kilogram of heavy Hydrogen fuel can produce as much energy as $10^6 kg$ of fossil fuel.

A plasma generation, insulated confinement and heating of ultra-high temperature steady-state plasma due to Bass [5] is employed in the current study. The closest prior related devices are the optical plasmotron by Raizer and the freely floating plasma filament of Kapitza and, in the field of migmas rather than plasma, and the Poissor of Farnsworth. Mitarai et al. investigate $D - {}^3He$ spherical and medium aspect ratio tokamak reactors [6]. The Bass Plasmasphere was designed to handle temperatures that are an order of magnitude greater than the aforementioned devices.

Mertogul and Krier studied non-equilibrium effects in LSP simulations [1]. Their work provides a comprehensive review of models. Their work on describing LSP theoretically is the most comprehensive and systematic work to date. However, their model is not appropriate for the optimization required in the present work. The model presented in this paper is generalized for any gas of interest. Further details are given in the thorough review given by Keefer [3].

Laser-spark experiments have been reported by many authors including Bekefi [7], Hughes [8], Raizer [9], and Ready [10]. Raizer's device called an "optical plasmatron" creates a fireball that can be maintained indefinitely by the use of lasers after its initial creation by spark-lasers. The experiments of Kapitza et al. have demonstrated that very similar results can be obtained by means of microwaves rather than laser beams [11].

This study reports a theoretical model to design a plasma with desired characteristics based on the relationship between ambient cold gas pressure, pulsed energy, pulsed laser wavelength, spark focal spot radius, supply laser wavelength and power, plasma transparency, bremsstrahlung loss, and any subsequent pressure increase ratio (or wavelength increase ratio) for further heating beyond mere steady-state maintenance. Previous investigations have not attempted to demonstrate theoretical plasma temperatures in the range of $10^9 K$ that is predicted in this study. Our model predicts far reduced power requirements, thus allowing for continuous operation similar to Kapitza's work and unlike the Tokamak reactors that operate on short pulses.

BACKGROUND

The proposed method to produce the laser-spark fireball is by an experiment in which the apparatus due to Bass [5] is employed, shown schematically in Figure (1). The following figures are important in understanding the analytical model. Figure (2) shows the important micro-physical principles upon which the operability is based. Below the main component is a graph to assist in explaining the theoretical principles underlying the operability. Special attention should be paid to the successful prediction by Oh and Bass [12, 13] that provides the insulation double-layer design, which was later confirmed experimentally by Eliezer and Ludmirsky [14].

Selected nomenclature for figures:

10,	A high-strength hydrogetic-gas-tight hollow chamber or pressure vessel (capable of holding e.g. 25 atmospheres pressure)
5,	Reservoir of high-pressure fluid medium (e.g. hydrogenic gas such as deuterium);
12,	Bubble-centering means (e.g. jet, sized by Stokes' Law)
20,	A selected high-pressure compressible fluid ambient filling medium (e.g. H or D or He gas)
31,	The fully ionized, neutral hot plasma created by explosion of 30 (e.g. a "laser spark") or expansion of 30
70,	Langmuir-Debye sheath or generalized Farnsworth-Kapitza effect boundary layer, including an eletrostatic double layer composed of 71 and 73
71,73	Layer of excess negative (alternatively positive) charge
90,	Exterior channel(s) for cooling fluid medium circulation
91,	A cooling fluid medium (e.g. water)
93,95,97,	Ambient fluid cooling-and circulation means (e.g. pump and refrigerator)
100,	Negative energy level
101,	Trapped ion(s) (in "potential well")
110,	Positive energy level
111,	Trapped electron(s)

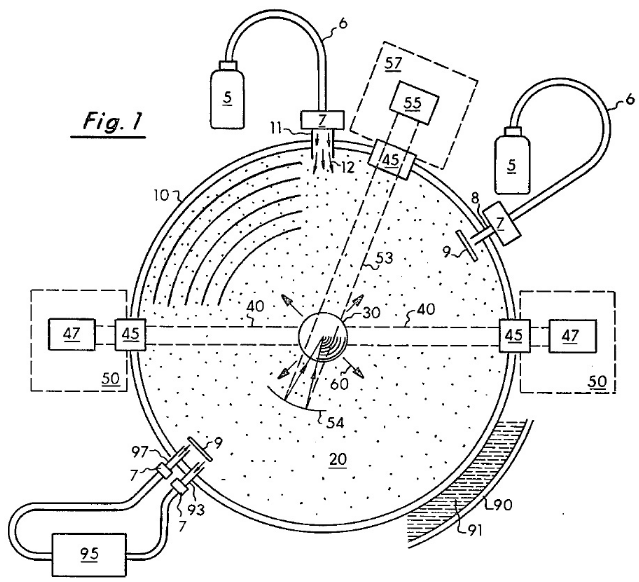

FIGURE 1. Cross-sectional schematic view of the Plasmasphere.

Figures (3) and (4) show how the optics work. For a complete understanding of this device the reader is referred to the US Patent for further details and figures [5].

Procedures associated with the operation are briefly reviewed here to elucidate the following generalized Bass model. We define state 0 as the ambient temperature $T_0 = 298K$ and $P_0 = 1atm$. The temperature at the critical point is defined by the ionization temperature $T_1 = T_i$, where the subscript refers to state 1. After the plasma has been established at state 1, further heating may be performed in either of two cases, namely isochoric heating and isobaric heating. For the isochoric case,

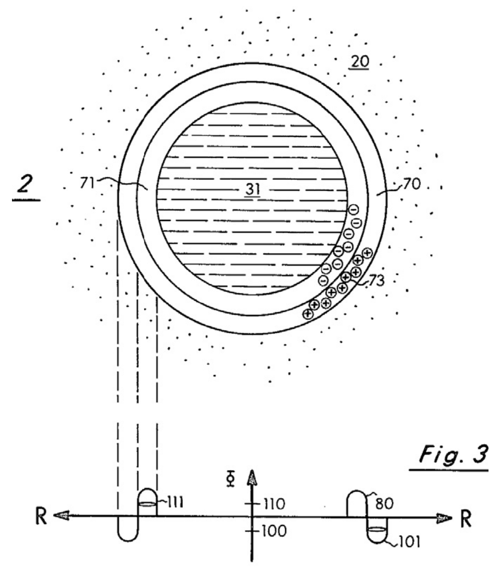

FIGURE 2. Expanded view of the central portion of the Plasmasphere.

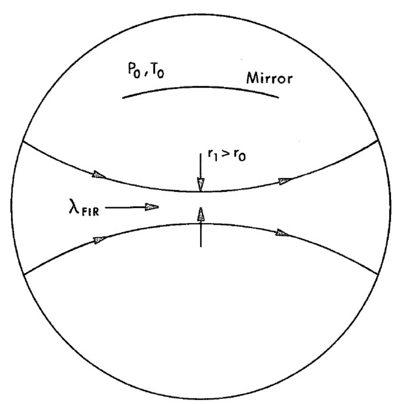

FIGURE 3. Plasmasphere schematic.

Fundamental Equations

The ideal gas law, or equation of state, provides the initial pressure P_0 by,

$$P_0 = n \cdot k \cdot T_0 = (N_0/V_0) \cdot k \cdot T_0 \tag{1}$$

where $n = N_0/V_0$ is the initial molecular number density, $k = 1.3807 \times 10^{-23} J \cdot K^{-1}$ is Boltzmann's constant, $T_0 = 288.2K$ is the initial temperature, and N_0 is the initial number of molecules in V_0, which is the initial volume at T_0. The initial state is generally denoted by the subscript "0". We let Z_g be the number of atoms per molecule and Z_e be the number of electrons per initial atom.

The energy E_p is required to dissociate and fully ionize all N molecules to atoms from molecules that contain Z_g atoms and expand the initial volume V_0 to a pressure-equilibrium plasma volume V_1 at temperature T_1. Therefore, V_1 is the final plasma volume and T_1 is the final plasma temperature. The pressure for the final plasma is deliberately set equal to the initial ambient gas pressure, i.e.,

$$P_1 = (Z_e \cdot Z_g \cdot N_0/V_1) \cdot k \cdot T_1 = P_0. \tag{2}$$

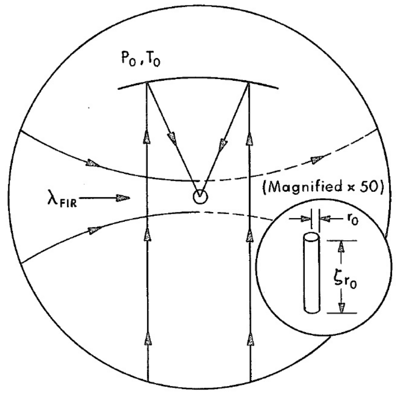

FIGURE 4. Parts description for laser-spark apparatus.

Internal energy U is defined as

$$U = P \cdot V/(\gamma - 1). \tag{3}$$

γ is the ratio of specific heats, which is defined as $\gamma_d = 7/5$ for diatomic gases, $\gamma_m = 5/3$ monotomic gases, and $\gamma_1 = 5/3$ for fully ionized plasma. γ_g is general such that $g = m, d$ depending on the gasses used. The change in internal energy is given by,

$$\begin{aligned} \Delta U &= P_1 \cdot V_1/(\gamma_1 - 1) - P_0 \cdot V_0/(\gamma_g - 1) \\ &= N_1 \cdot k \cdot T_1/(\gamma_1 - 1) - N_0 \cdot k \cdot T_0/(\gamma_g - 1). \end{aligned} \tag{4}$$

From equations (1), (2), and (3), conservation of energy gives us

$$\eta \cdot E_p = \Delta U + P_0 \cdot \Delta V + N_0 \cdot E_i, \tag{5}$$

where η is the efficiency of absorption of E_p, the total pulse energy. E_i is the energy of both molecular dissociation and ionization of resultant atoms and

$$P_0 \cdot \Delta V = P_0(V_1 - V_0) = P_1V_1 - P_0V_0 = Z_e \cdot Z_g \cdot N_0kT_1 - N_0kT_0. \tag{6}$$

Equation (5) can be rewritten as

$$\eta \cdot E_p = Z_e \cdot Z_g \cdot \beta N_0kT_1 - \Gamma N_0kT_0 + N_0 \cdot E_i, \tag{7}$$

where,

$$\begin{aligned} \beta &= 1/(\gamma_1 - 1) + 1 = 5/2 \\ \Gamma_d &= 1/(\gamma_d - 1) + 1 = 7/2 \\ \Gamma_m &= 1/(\gamma_m - 1) + 1 = 5/2. \end{aligned} \tag{8}$$

We define the dimensionless parameters:

$$\begin{aligned} \kappa_i &= E_i/(k \cdot T_0), \\ \theta &= T_1/T_0, \\ \alpha_p &= E_p/(N_0 \cdot k \cdot T_0). \end{aligned} \tag{9}$$

Therefore, equation (7) can be rewritten as

$$\eta \cdot \alpha_p = Z_e \cdot Z_g \cdot \beta\theta - \Gamma_g + \kappa_i. \tag{10}$$

GENERALIZED BASS MODEL

The spark-laser fireball is to be designed with desired qualities based on the following model.

If the spark laser pulse duration is t_p, for times $t_p < 10ns$ (nanoseconds), it is well known [7, 8, 9, 10] that the gas ionization breakdown threshold intensity is

$$I_0 = k_1 / \left[\lambda_p^2 \cdot P_0 \cdot t_p\right], \tag{11}$$

where k_1 is a material constant, λ_p is the spark laser wavelength, and P_0 is the ambient gas pressure.

The plasma resonant absorption frequency is given by

$$\nu = c/\lambda_{sup} = k_2\sqrt{N_{ec}}, \tag{12}$$

k_2 is a material constant, $c = 2.9979 \times 10^8 m/s$ is the speed of light, λ_{sup} is the wavelength of the Far Infrared supply laser or any general power supply, and N_{ec} denotes the critical electron density as

$$N_{ec} = Z_{e0} \cdot Z_{g0} \cdot N_0/V_1. \tag{13}$$

V_1 is the final plasma volume formed by complete ionization of the N_0 original molecules with Z_{g0} atoms and Z_{e0} electrons per molecule. The plasma equation of state in equation (2) can be rewritten using the electron density similar to equation (13), $N_e = Z_e \cdot Z_g \cdot N/V$, and plasma temperature T as

$$P = (Z_e \cdot Z_g)^{-1} \cdot N_e \cdot k \cdot T. \tag{14}$$

According to Guenther [15], the transmissivity is given by,

$$\eta = 1 - \exp(-K_\nu \cdot l) \tag{15}$$

where

$$K_\nu = k_3 \cdot N_e^2/(\nu_l^3 \cdot T^{1/2}). \tag{16}$$

l is the length of the optical path, ν_l is the frequency of light, and $k_3 = 3.68 \times 10^{-2} m^5 \cdot k^{1/2} \cdot s^{-6}$. Initially, $N_e = N_{ec}$ is employed in equation (15) for the supply-laser-design using λ_{sup}. We assume a uniform laser pulse shape with pulse duration t_p. The initially-ionized volume is a cylinder with radius r_0, the initial focal spot radius, and length $l = \zeta \cdot r_0$ ($\zeta > 1$) to give the volume $V_0 = l \cdot \pi \cdot r_0^2 = \zeta \cdot \pi r_0^3$. The pulse intensity is given by,

$$I_{0p} = E_p/(\pi \cdot r_0^2 \cdot t_p). \tag{17}$$

From equation (10), we solve for θ,

$$\theta = (\eta \cdot \alpha_p + \Gamma - \kappa_i)/(\beta \cdot N_1/N_0). \tag{18}$$

By design, we choose the final pressure-equilibrium plasma radius to be $r_1 = \lambda_{sup}$. We define the unknown variable

$$x = r_0/r_1 < 1. \tag{19}$$

Therefore, we can express r_0 in terms of the unknown by,

$$r_0 = x \cdot \lambda_{sup}. \tag{20}$$

The initial and final volumes are rewritten as

$$V_0 = \pi\zeta \cdot \lambda_{sup}^3 \cdot x^3, \tag{21}$$

$$V_1 = (4/3)\pi\lambda_{sup}^3. \tag{22}$$

From equations (1) and (21), we rewrite the initial pressure as

$$P_0 = N_0 \cdot k \cdot T_0/(\pi \cdot \zeta \cdot \lambda_{sup}^3 \cdot x^3) \tag{23}$$

and the critical electron density becomes

$$N_{ec} = 3Z_{e0} \cdot Z_{g0} \cdot N_0/(4\pi \cdot \lambda_{sup}^3) = (c/[k_2 \cdot \lambda_{sup}])^2. \tag{24}$$

To obtain N_0 in terms of λ_{sup} alone and in terms of λ_{sup} and $P_0 \cdot x^3$, we use equations (1), (23), and (24) to get

$$N_0 = \frac{4\pi\lambda_{sup}}{3Z_{e0} \cdot Z_{g0}} \cdot \left(\frac{c}{k_2}\right)^2 = \frac{P_0\pi\zeta \cdot \lambda_{sup}^3}{k \cdot T_0} \cdot x^3 \tag{25}$$

and another useful relationship, namely

$$P_0 \cdot x^3 = 4k \cdot T_0/(3\zeta \cdot \lambda_{sup}^2 \cdot Z_{e0} \cdot Z_{g0}) \cdot (c/k_2)^2. \tag{26}$$

For later convenience, equations (25) and (26) yield the following relationship,

$$N_0 \cdot k \cdot T_0 = (4\pi k \cdot T_0 \cdot \lambda_{sup}^2)/(3Z_{e0} \cdot Z_{g0}) \cdot (c/k_2)^2. \tag{27}$$

From equations (9), (17), (20), and (27), we obtain

$$\alpha_p = k_1\pi\lambda_{sup}^2 x^5/(\lambda_p^2 \cdot P_0 \cdot x^3 \cdot N_0 \cdot k \cdot T_0) = \varepsilon_p x^5, \tag{28}$$

where the pulse density from equation (17) is assumed to be $I_{0p} = I_0$ and ε_p is given by

$$\varepsilon_p = (3Z_{e0} \cdot Z_{g0})^2\zeta \cdot k_1 \cdot \lambda_{sup}^3 \cdot (k_2/c)^4/(4\lambda_p \cdot k \cdot T_0)^2 \tag{29}$$

from equations (11), (26), and (25).

The pressure from equations (1) and (2) give us

$$P_0 = (N_0/V_0) \cdot k \cdot T_0 = (N_1/V_1) \cdot k \cdot T_1. \tag{30}$$

The temperature ratio θ in equation (18) is rewritten using equation (22) as

$$\theta = T_1/T_0 = 4N_0/(3\zeta \cdot N_1 \cdot x^3). \tag{31}$$

Combining equations (18) and (31), equation (28) can be rewritten as

$$\eta \cdot \varepsilon_p \cdot x^8 + (\Gamma - \kappa_i) \cdot x^3 - 4\beta/(3\zeta) = 0. \tag{32}$$

Equation (32) needs to be written with η in terms of x. First, we use equations (13), (16), and (21) to obtain equation (33). Here, V_0 is substituted by V_1 in equation (13) since this describes the second step as described above.

$$K_v = \frac{k_3\,(Z_{e0} \cdot Z_{g0} \cdot N_0)^2\,\lambda_p^3}{(c \cdot \lambda_{sup}^2)^3 T_1^{1/2}(\pi \cdot \xi)^2 x^6} \tag{33}$$

$$K_v \cdot l = K_v \cdot \zeta \cdot r_0 = k_4/x^5 \tag{34}$$

where

$$k_4 = \frac{k_3\,(Z_{e0} \cdot Z_{g0} \cdot N_0)^2\,\lambda_p^3}{\zeta \cdot c^3 \cdot T_1^{1/2}\pi^2\lambda_{sup}^5}. \tag{35}$$

Using equation (15), equation (32) becomes

$$\left[1 - \exp(-k_4/x^5)\right] \cdot \varepsilon_p \cdot x^8 + (\Gamma - \kappa_i) \cdot x^3 - 4\beta/(3\zeta) = 0. \tag{36}$$

We can employ Newton's method to solve equation (36) iteratively, i.e.,

$$x_{n+1} = x_n - f(x_n)/f'(x_n) \tag{37}$$

with an estimated initial value x_0. However, in practice, we found the Roe procedure to converge rapidly to a self-consistent solution of equation (36). The unknown quantities r_0, θ, P_0, E_p are determined from equations (20), (31), (26), respectively, and T_1 is obtained from $\theta \cdot T_0$. Assuming $I_{0p} = I_0$, equations (11) and (17) give us,

$$E_p = k_1 \pi r_0^2 / \left(\lambda_p^2 \cdot P_0\right). \tag{38}$$

For the next step, we define the temperature T_3 as a prescribed value and we let ξ be unknown, where

$$0 < \xi < 1 \tag{39}$$

and by definition, in the third step,

$$r_3 = r_1/\xi = \lambda_{sup}/\xi \tag{40}$$

whence

$$V_3 = V_1/\xi^3 \tag{41}$$

$$N_{e3} = \xi^3 \cdot N_{e1}. \tag{42}$$

The relationships for pressure during the second and third steps are similar to equation (2), i.e.,

$$P_2 = N_2 \cdot k \cdot T_1/V_1 = P_3 = N_2 \cdot k \cdot T_1/V_1. \tag{43}$$

Therefore, we have

$$\xi^3 \cdot N_{e1} \cdot T_3 = N_{e3} \cdot T_3 = N_{e1} \cdot T_2 \tag{44}$$

which gives $T_2 = \xi^3 \cdot T_3$.

The frequency of the supply laser or microwave ν is taken to be

$$\nu_{sup} = \nu_{mic}. \tag{45}$$

From equations (16), (12), and (42) we have

$$K_\nu = k_7 \cdot \xi^6, \tag{46}$$

where

$$k_7 = k_3 N_{e1}^2 / \left[(c/\lambda_{sup})^3 T_3^{1/2}\right] \tag{47}$$

The final transparancy to the microwave is defined by the absorption efficiency factor given by

$$\eta_{mic} = 1 - \exp(-2K_\nu \cdot r_3) = 1 - \exp(-2k_7 \cdot \lambda_{sup} \cdot \xi^5) \tag{48}$$

where

$$\xi = \left(\left[\log(1 - \eta_{mic})^{-1}\right] / [2k_7 \lambda_{sup}]\right)^{1/5}. \tag{49}$$

We may now express P_2 in terms of P_0 from using equations (2), (14), and (47) to obtain

$$P_2 = \frac{\xi^3 \cdot T_3 \cdot Z_{e1} \cdot Z_{g1} \cdot N_{e2}}{T_1 \cdot Z_{e2} \cdot Z_{g2} \cdot N_{e1}} \cdot P_0 \tag{50}$$

The bremsstrahlung loss decreases during the final expansion. The power is given by,

$$P_{Br,1} = C \cdot N_{e1}^2 \cdot (T_2)^{1/2} \cdot V_1. \tag{51}$$

If T_2 is expressed in keV, then $C = 5.354 \times 10^{-37} W \cdot m^3/k^{1/2}$. The second step has, from equation (39),

$$P_{Br,2} = C \cdot N_{e3}^2 \cdot (T_3)^{1/2} \cdot V_3 = C \cdot (N_{e1}\xi^3)^2 \cdot (T_2/\xi^3)^{1/2} \cdot (V_1/\xi^3) = \xi^{3/2} \cdot P_{brem,1} < P_{Br,1}. \tag{52}$$

To find ξ, we employ equation (48) and assume the final transparency to the microwave to be defined by the absorption efficiency factor,

$$\eta_{mic} = 10^{-3}. \tag{53}$$

To find x, we assume that $\zeta = 10$, which agrees well with numerous measured experiments and is satisfactorily accurate for the calculations in the next section. Another reason is that equation (36) is relatively insensitive to variations in ζ near the nominal value, $\zeta = 10$.

The microwave power P_{mic} can be found from $\eta_{mic} \cdot P_{mic} = P_{Br,2}$, i.e. by equation (52) to give us

$$P_{mic} = P_{Br,1} \cdot \left(\xi^{3/2}/\eta_{mic}\right). \tag{54}$$

SIMULATIONS

Hydrogen Case

The ionization potential for an H-atom is $T_i = 13.6eV = 1.578 \times 10^5 K$. For two atoms we have $27.2eV$ and, using the dissociation-potential of $3.6eV$, we have $E_i = 30.8eV$ for H_2 molecules. Therefore, we find that

$$\kappa_i = E_i/(k \cdot T_0) = 1.241935 \times 10^3. \tag{55}$$

From equation (8), we have $\Gamma = 3.5$ and $\beta = 2.5$. For Hydrogen, $k_1 = 1.49 J^2 \cdot m^{-3}$ from theoretical and experimental results [7, 8, 9, 10]. Unless otherwise stated, these references are used for obtaining unknown model parameters. The remaining parameters are: $Z_{e0} = 1$, $Z_{g0} = 2$, $Z_{e1} = 1$, $Z_{g1} = 2$, $Z_{e2} = 1$, $Z_{g2} = 2$, $k_2 = 8.98 s^{-1} \cdot m^{3/2}$. Note that in the case when atoms recombine in step 1 or 2, the Z_{gi} coefficients change.

Helium Case

As inert gases do not have dissociation processes, the ionization processes are relatively easy. The first ionization energy for Helium is $24.6eV$ and $\kappa_i = 9.914 \times 10^3$. The reaming parameters needed are: $Z_{e0} = 2$, $Z_{g0} = 1$, $Z_{e1} = 2$, $Z_{g1} = 1$, $Z_{e2} = 2$, $Z_{g2} = 1$, $k_1 =$[need value], $k_2 =$[need value].

Results and Discussion

CONCLUSIONS

- The generalization of the Bass equations permits its use with arbitrary gasses. The Hydrogen and Helium simulations illustrate the use and make predictions that require future experiments to validate. Although validation at this time is not possible, the proposed analytic model is derived entirely from fundamental and well established relationships.
- For the desired temperature, plasma ball radius, and density, the energy requirement is well within a reasonable budget.
- The minimum fireball diameter for Helium is $1cm$ and the maximum diameter is limited by budget.

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