

EXHIBIT A

Submitted to 1975 Awards for Essays on Gravitation

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WHICH METRIC GRAVITATIONAL THEORY BEST FITS
MEASURED GEOPHYSICAL, ASTROPHYSICAL AND COSMOLOGICAL DATA?
(Statistical Parameter Estimation and Incorporation
Of the Einstein-Milne-Sciama Interpretation
Of the Berkeley-Mach Principle in a
Jordan-Brans-Dicke-Bergmann-Wagoner Type of
Scalar-Tensor Field Extension of General Relativity)

by

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SUMMARY

Abundant geophysical, astrophysical and cosmological evidence is explainable most readily by consequences of an Einstein desideratum for Mach's Principle: Cavendish parameter G should increase as one approaches a massive body (exact opposite of Brans-Dicke hypothesis, but a feature of Einstein-Hlavaty asymmetric, non-metric theory). A corresponding physically viable (95% probability) scalar-tensor theory predicts (as did Milne, in exact opposition to Dirac) that G increases in time, and that complete gravitational collapse is impossible: black holes do not exist; quasars are merely compact pulsating stars. A new critical test aboard the Helios satellite and nuclear applications are proposed.

CONTENTS

1. Geophysical anomalies explainable by an increasing G .
2. Astrophysical evidence of necessity for an improved gravitational theory ($\gamma > 1$, $q < \frac{1}{2}$).
3. A new Milne-type (as opposed to Dirac-type) of scalar-tensor gravitational field theory compatible with all physical evidence (and thus now the only viable theory).

ERRATA AND CORRIGENDA

1. Exhibit A

- a. Page 2, bottom line: strike out first factor 2.
- b. Page 8, line 8: insert [Our Maximum Likelihood estimate is $\gamma = 1.02 \pm 0.01$.]
- c. Page 14, next to bottom line: version²².

2. Appendix 1

- a. Abstract, line 7: replace "weakens" by "changes" and ± 0.06 by ± 0.01 ; in next line replace 60% by 95%.
- b. Page 3: delete factors $(1/N)$ from (6) and (7),
- c. Page 4, equation (8): replace $\pm 0.20 > 1.01$ by $\pm 0.06 > 1.15$; in (10): replace $\pm 0.088 > 0.92$ by $\pm 0.062 > 0.95$.
- d. Page 5, equation (11): replace $\pm 0.059 > 0.96$ by $\pm 0.018 > 1.004$; equation (12): replace $\pm 0.029 > 0.98$ by $\pm 0.011 > 0.99$; equation (13): replace $\pm 0.06 > 0.96$ by $\pm 0.01 > 1.01$; two lines below (13) replace 60% by 95%.
- e. Page 8: replace $\pm 0.20 > 1.01$ by $\pm 0.06 > 1.15$.
- f. Page 10: replace $\pm 0.088 > 0.92$ by $\pm 0.062 > 0.95$.
- g. Page 11: replace $\pm 0.059 > 0.96$ by $\pm 0.018 > 1.004$.
- h. Page 12: replace last column by 0.06, 0.015, 0.062, 0.018 and replace $\pm 0.029 > 0.98$ by $\pm 0.011 > 0.99$.
- i. Page 14: replace ± 0.055 by $\pm 0.011 > 1.006$.

3. Appendix 3A

- a. Page 2, equation (7): insert minus sign before right-hand side.

4. Appendix 3B

- a. Insert reference 11:
 - A. Einstein, "Stationary systems with spherical symmetry consisting of many gravitating masses," Annals of Mathematics, Ser. 2, vol. 40 (1939), pp. 922-936.

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1. GEOPHYSICAL ANOMALIES EXPLAINABLE
BY AN INCREASING G.

Most of the quantitative observational evidence which has been adduced¹ for the Dirac hypotheses that $(\dot{G}/G) < 0$ either depends on unverified approximate calculations concerning complex phenomena (e.g., lunar tidal friction) or else concerns data at the limit of meaningful measurements, and so cannot be regarded as fully conclusive. In fact, Dicke¹ admits that ". . . the foregoing discussion cannot be marshaled as evidence for a gradual decrease in the gravitational constant."

However, all qualitative phenomena unambiguously associable with $(\dot{G}/G) \neq 0$ support the Milne hypothesis $(\dot{G}/G) > 0$.

1.1. Anomalous terrestrial core heat flux explainable by shrinkage of core radius.

Dicke¹ has surveyed the evidence for heat production in the oceanic and continental crusts, and finds that "the large value for heat flow through the ocean floor is something of a mystery. If the foregoing assumptions concerning the distribution of radioactivity are correct, this heat cannot arise in the oceanic crust but must have its origin deep in the mantle." From all sources Dicke computes "a total of 13 to 15 ergs/cm²sec. This is to be compared with an observed average flux of 50 ergs/cm²sec. The agreement is not particularly good." Dicke posits ad hoc a complicated mechanism by which weakening gravitation would result in a cooling interior.

More straightforward is the hypothesis that increasing G, and a consequently shrinking core, represents in effect work done compressing the core by the expanding universe, which thereby releases the discrepant 35 ergs/cm²sec observed. To see this, consider the Newtonian n-body gravitational problem

which I have shown² can be written as (using $\dot{} = d/dt$)

$$(1) \quad \ddot{\mathbf{x}} = GV(\mathbf{x}), \quad V(\mathbf{x}) \equiv V(-\mathbf{x}) \equiv V(\mathbf{x}/|\mathbf{x}|)/|\mathbf{x}| > 0$$

where \mathbf{x} is a 3n-vector. The conservation of energy for a gravitationally bound system is, for G constant,

$$(2) \quad E = \frac{1}{2}|\dot{\mathbf{x}}|^2 - GV(\mathbf{x}) \equiv -|E_0|.$$

But for variable G ,

$$(3) \quad E = \frac{1}{2}|\dot{\mathbf{x}}|^2 - GV(\mathbf{x}) \equiv -|E_0| - \int_0^t \dot{G}V dt.$$

For isolated particles which interact only gravitationally, $(\dot{G}/G) > 0$ implies a loss of total energy, i.e., the particles orbit nearer their center of mass. But for a solid body, shrinkage would involve compression, friction, heating and loss of heat to the crust by convection and conduction.

1.2. Anomalous secular acceleration of terrestrial rotation virtually proves Earth's shrinkage.

According to a UPI release of March 21, 1975 "Dr. Paul Lowman of the Goddard Space Flight Center . . . discounts the theories the continents have grown in land size. He believes they are shrinking."

Let the 3-vector $\vec{\omega}$ denote the Earth's angular velocity and J its inertia tensor. If $(\dot{G}/G) > 0$, i.e., if the Earth is shrinking uniformly then (cf. (7) below)

$$(4) \quad \dot{J} = -2\varepsilon J, \quad 2\varepsilon = (0.2)(\dot{G}/G), \quad (\dot{} = d/dt)$$

while Euler's equations are

$$(5) \quad J\dot{\vec{\omega}} + \vec{\omega} \wedge J\vec{\omega} = -\dot{J}\vec{\omega}.$$

Now $(\vec{\omega} \cdot J\vec{\omega}) \cdot = -2(\vec{\omega} \cdot \dot{J}\vec{\omega}) = 2\varepsilon(\vec{\omega} \cdot J\vec{\omega})$, or

$$(6) \quad \vec{\omega} \cdot \dot{\vec{\omega}} = (\omega_0 \cdot \dot{\omega}_0) e^{2\epsilon t}, \quad (0 \leq t < +\infty).$$

Hence the kinetic energy of rotation must be increasing, i.e., the Earth experiences a secular acceleration of its rate of rotation.

The analysis by Currott³ and Dicke⁴ of five eclipses between 1062 B. C. and 71 A. D. shows that Earth's $|\vec{\omega}|$ decreases relative to planetary periods by $(15.9 \pm 0.7) \times 10^{-11}$ parts per year. Subtracting⁵ all known influences (e.g., tidal deacceleration, an acceleration due to rise in sea level, and isostatic recovery of the geoid) still leaves "a residual unexplained acceleration of the Earth's rotation between 4×10^{-11} and 10×10^{-11} parts per year."

According to G. Hess⁶ and C. Murphy⁷

$$(7) \quad \epsilon = -\dot{r}/r = 0.1(\dot{G}/G)$$

Now the observations cited prove that, in (6), $\epsilon > 0$; hence, in (7), $(\dot{G}/G) > 0$. (Even if the quoted residual acceleration is not very accurate quantitatively, its qualitative correctness would prove conclusively that $(\dot{G}/G) > 0$.)

1.3. Anomalous solar neutrino flux explainable by Sun's shrinkage.

The Sun's radiation rate or luminosity L can be shown⁸ to be proportional to the seventh or eighth power of G and also to be a function of its mass M , radius R , and central temperature T :

$$(8) \quad L \propto G^7 M^5 T^{1/2} / R^7.$$

When G is assumed constant, known nuclear processes can be used to establish a quantitative model⁹ of the Sun's internal reactions and temperature T corresponding to the observed values of L , M , and R ; in turn, this model predicts a definite solar neutrino flux at 1AU. To quote Trimble and Reines¹⁰: "The conflict between observation and theoretical prediction of the flux of electron neutrinos from the sun has advanced in the past year from being merely

difficult to understand to being impossible to live with." A theoretical loop-hole, which they describe as a "DESPERATE MEASURE", is the ad hoc postulate of Fowler that the present central solar temperature T is below its average value. According to the theory to be developed here, no ad hoc hypotheses are needed. The shrinkage of the sun would provide, in effect, an energy source additional to nuclear transmutation, which would necessitate lowering the postulated central temperature T in order to be consistent with the observed L .

2. ASTROPHYSICAL EVIDENCE OF NECESSITY FOR AN IMPROVED GRAVITATIONAL THEORY ($\gamma > 1$, $q < \frac{1}{2}$).

2.1. Both General Relativity (GR) and the Jordan-Brans-Dicke (JBD) extension fail to incorporate the Berkeley-Mach Principle as interpreted by Einstein, Milne, and Sciama.

It has been shown by Dicke¹ that a scalar-tensor theory in which G varies is equivalent to standard GR but with variable inertial masses. Einstein¹¹ wanted to prove that "the inert mass . . . increases when ponderable masses approach the test body." This is equivalent to assuming that G increases as one test body approaches a ponderable mass.

The same effect arises naturally in the asymmetric, non-metric Einstein-Hlavaty theory, where, in the vicinity of an electron or proton of charge e ,

$$(9) \quad G = \frac{G}{1 - \frac{\mu e^2}{r^4}}, \quad (\mu > 0)$$

as can be seen from Hlavaty¹² (pp. 42, 185, 161, 202, 205) and Adler, Bazin and Schiffer¹³ (pp. 398-399). Thus there is a radius $r_\mu = (\mu e^2)^{\frac{1}{4}}$ such that*

$$(10) \quad G \rightarrow +\infty \quad \text{as} \quad r \rightarrow (r_\mu + 0).$$

*In Hlavaty's notation, $G = 1$, $\mu = 1$; presumably $\mu = L^3/m_e c^2$, $L = (\hbar G/c^3)^{\frac{1}{2}} = 1.6 \times 10^{-33}$ cm; $e^2/m_e c^2 = 2.8 \times 10^{-13}$ cm; i.e., $r_\mu = 1.8 \times 10^{-28}$ cm.

In the initial geometrical version of his scalar-tensor theory, Jordan¹⁴ allowed a dimensionless coupling coefficient ω of arbitrary sign, but in applications to physics he followed the idea of Dirac that $(\dot{G}/G) < 0$ and so assumed $\omega > 0$. In the Brans-Dicke-Bergmann-Wagoner theory it is explicitly assumed that

$$(11) \quad -3/2 < \omega < +\infty,$$

where agreement with GR is obtained as $\omega \rightarrow +\infty$; in fact, a key result of JBD is

$$(12) \quad \frac{\dot{G}}{G} = - \frac{8\pi G_0 \rho_0 t_0}{(2\omega + 3)},$$

since JBD follow Dirac in assuming $(\dot{G}/G) < 0$, they must also assume (11). It turns out, however, that (11) excludes any such possibility as (10).

If, instead, JBD had assumed

$$(13) \quad -\infty < \omega < -2 < -3/2 < 0$$

then they would not only have found $(\dot{G}/G) > 0$, but (as we shall show) they also would have found (10) as well. However, in many articles and a book, Dicke¹ has stressed repeatedly the physical consequence of (11), including the fact that

$$(14a) \quad (\text{PPN:}) \quad \gamma = \frac{\omega + 1}{\omega + 2} = 1 - \frac{1}{\omega + 2} < 1 \quad (-3/2 < \omega < +\infty).$$

In contrast, in the present theory

$$(14b) \quad (\text{PPN:}) \quad \gamma = \frac{|\omega| - 1}{|\omega| - 2} = 1 + \frac{1}{|\omega| - 2} > 1 \quad (2 < |\omega| < +\infty).$$

Hence it seems fair to emphasize that the present theory is not a special case of JBD: despite formal similarities, it is an entirely different theory, having radically different observational bases ($\gamma > 1$, $q < \frac{1}{2}$), different physical

motivations and interpretations, and a different parameter-regime of validity. (Nevertheless, if one makes the [trivially simple] transformation $|\omega| = -\omega$, followed by the [radical] change from the parameter region (13) to the disconnected region (11), one recovers JBD; for this reason, all known formal results^{15,16} in JBD can be readily transformed into results in the present theory.) We shall symbolize (10) by $(\delta G/G) > 0$. Hence we have

(15)	Present theory	JBD
(15a)	$-\infty < \omega < -2$	$-3/2 < \omega < +\infty$
(15b)	$(\dot{G}/G) > 0$ (MILNE)	$(\dot{G}/G) < 0$ (DIRAC)
(15c)	$(\delta G/G) > 0$ (EINSTEIN)	$(\delta G/G) < 0$ (DICKE)
(15d)	(PPN:) $\gamma \approx 1$	(PPN:) $\gamma < 1$
(15e)	recover GR as $\omega \rightarrow -\infty$	recover GR as $\omega \rightarrow +\infty$

The key result in E. A. Milne's¹⁷ interpretation of Mach's Principle is

$$(16) \quad G \propto c^3 t / M$$

where c denotes light-speed and M denotes the (constant) mass of the visible universe (approximately a sphere of radius ct as cosmic time t increases indefinitely). Also Sciama's interpretation of Mach's Principle yields

$$(17) \quad G \propto c^2 R / M,$$

whence the assumptions

$$(18a) \quad M = \text{constant}, \quad (18b) \quad R \propto ct \quad (t \rightarrow +\infty)$$

put Sciama's interpretation in exact agreement with that of Milne. In the present theory we shall prove (18a,b) as well as (17), hence deriving Milne's controversial, isolated and usually ignored but central result (16) as a result of a covariant metric theory derived from a Lagrangian variational principle.

2.2. Observed hyperbolicity of the universe (Sandage, 1974; Gunn, 1974) physically contradicts both GR and JBD. Anomalous red-shifts of quasars suggest improvability of GR.

Let R denote the expanding radius of the visible universe, containing a total mass M , and let

$$(19) \quad r_{LS} = 2GM/c^2$$

denote the Laplace-Schwartzschild "black-hole" radius of the mass M . Let

$$(20) \quad q \stackrel{d}{=} -(\ddot{R}/R)/H^2, \quad H \stackrel{d}{=} \dot{R}/R$$

denote the usual cosmological parameters. In Appendix 2, we prove the new rigorous result

$$(21) \quad R = [(1 - 2q)/2q] r_{LS}, \quad q = q(t) \rightarrow 0, \quad t \rightarrow +\infty,$$

which holds in either a Newtonian or a Friedmannian hyperbolic [$q < \frac{1}{2}$] cosmology. This result yields a physical absurdity: that although $R(0) = 0$, there is a cosmic time $t_{LS} > 0$ such that $R(t) > r_{LS}$ for $t > t_{LS}$, i.e., matter escapes from a black hole! This hitherto unnoticed paradox of GR can be avoided only by allowing (16); hence by (15) the observed hyperbolicity of the universe physically rules out GR and JBD and requires the present theory. Furthermore, it will be shown that the present theory naturally explains the apparently "anomalous" quasar red-shifts, without any need for ad hoc hypotheses.

2.3. The twenty-four known solar system experiments contradict both GR and JBD (to two standard deviations!)

In Appendix 1, we present a comprehensive statistical analysis of all twenty-four known examples of the four known classes of solar system experiments pertaining to metric gravitational theories. The mean value $\bar{\gamma}$ and its

standard deviation $\bar{\sigma}$ for the Eddington-Robertson (PPN) parameter γ are given by

$$(22) \quad \bar{\gamma} \pm \bar{\sigma} = 1.10 \pm 0.05 ,$$

i.e., if the observation errors are random and normal, then

$$(23) \quad (\text{PPN:}) \quad \gamma > 1 \text{ with about 95\% probability.}$$

Comparison of (22)-(23) with (14)-(15) provides further powerful evidence in favor of the present theory as opposed to GR or JBD: in the terminology of Will¹⁹, to two standard deviations the present theory is now the only "viable" metric gravitational theory.

3. A NEW MILNE-TYPE (AS OPPOSED TO DIRAC-TYPE) OF SCALAR-TENSOR GRAVITATIONAL FIELD THEORY COMPATIBLE WITH ALL PRECEDING PHYSICAL EVIDENCE (AND THUS NOW THE ONLY VIABLE THEORY)

3.1. Closed-form solution in standard coordinates for a static, spherically symmetric gravitational field: both Cavendish parameter G and anti-gravitational repulsion become infinite at an impenetrable Laplace-Schwarzschild-type (LS) radius r_{LS} . Complete gravitational collapse impossible: true black holes do not exist; quasars are merely compact stars. Definitive experimental test of the present theory proposed for performance onboard a Helios satellite.

Let (in units where $c = 1$)

$$(24) \quad r_{LS} = 2GM$$

$$(25) \quad \tilde{r}_{LS} = \alpha(|\omega|) \approx \left(\frac{|\omega| - 1}{|\omega| - 2} \right) r_{LS} \quad (|\omega| \rightarrow +\infty)$$

$$(26) \quad \tilde{\tilde{r}}_{LS} = \frac{\tilde{r}_{LS}}{\{1 - (|\omega| - 1)^{-[1 + 5/8|\omega|]}\}} > \tilde{r}_{LS} > r_{LS} > 0 ,$$

where the rational algebraic function $\alpha = \alpha(|\omega|) > 0$ is defined quite explicitly in Appendix 3. Here G denotes the value of G that would obtain in our solar

system's region of the galaxy if the Sun were absent. The actual value of G at a "distance" r from the Sun is given by

$$(27) \quad G = G \frac{(1 + \beta/r)^\delta}{(1 - \alpha/r)^\gamma}$$

where the positive [non-PPN] parameters $(\alpha, \beta, \gamma, \delta)$ are explicit functions of $|\omega|$, M and G such that, e.g.,

$$(28) \quad \alpha\gamma + \beta\delta \equiv \frac{MG}{(|\omega| - 2)} \frac{d}{r} \varepsilon ,$$

whence, as $r \rightarrow +\infty$,

$$(29) \quad G = G(1 + \frac{\varepsilon}{r} + \dots) .$$

As $|\omega| \rightarrow +\infty$, $\alpha \rightarrow r_{LS}$, $\beta/\varepsilon \rightarrow 0$, $\delta \rightarrow (2|\omega| - 2)^{-1}$, $\gamma \rightarrow (2|\omega| - 2)^{-1}$.

Also, clearly,

$$(30) \quad G \rightarrow +\infty \quad \text{as} \quad r \rightarrow (\tilde{r}_{LS} + 0) ,$$

which confirms Einstein's expectation (10). However, (27) has no direct physical meaning when there is only one test particle near the mass M ; but if there are two test particles near \tilde{r}_{LS} , then the ambient G for them is the unbounded G of (30), i.e., (in the first approximation) they attract like Newtonian particles with infinite G .

The geodesic equations of a test particle yield an equivalent Newtonian potential ϕ and radial force $-\phi'(r)$ corresponding to the present metric, from which we prove [Appendix 3] that

$$(31) \quad -\phi'(r) \begin{cases} \leq 0 \\ \geq 0 \end{cases} \Leftrightarrow r \begin{cases} \geq \tilde{r}_{LS} \\ < \tilde{r}_{LS} \end{cases}$$

and

$$(32) \quad -\phi' \rightarrow +\infty \quad \text{as} \quad \tilde{r}_{LS} > r \rightarrow (\tilde{r}_{LS} + 0) ,$$

that is, the gravitational interaction changes from attractive to repulsive near and just outside the radius \tilde{r}_{LS} ($\approx r_{LS}, |\omega| \rightarrow +\infty$) and actually becomes infinitely repulsive as that radius is approached (externally). Hence complete gravitational collapse is impossible.

The fact that black holes cannot exist is a rigorous corollary of the present theory. This is true even as $|\omega| \rightarrow +\infty$, i.e., as GR is recovered! Indeed, as shown by Rosen²⁰, this result could have been anticipated from GR, in which the non-physical nature of curves $r = \text{constant} < r_{LS}$ is apparent from the fact that they are simultaneously space-like and time-like. The standard comment that $R_{\lambda\mu\nu\sigma}$ is well-behaved at $r = r_{LS}$ is irrelevant, because what goes wrong there is global rather than local.

In the present theory, the red-shift of light emitted from the surface of a star of radius R as perceived at infinity is given by [Appendix 3]

$$(33) \quad z = \frac{\Delta v}{v} = -1 + \frac{(1 + \frac{\beta}{R})(|\omega| - 2)\delta}{(1 - \frac{\alpha}{R})(|\omega| - 2)\gamma} = \\ = r_{LS}/2R + \dots \quad (R \rightarrow +\infty).$$

Hence for compact stars every possible red-shift ($0 < z < +\infty$) can occur, depending upon how close the star's radius R is to its \tilde{r}_{LS} . (By (31)-(32), $R > \tilde{r}_{LS}$ always.) [Whereas, in GR, $z < 2$ for⁵ any possible star.] We would expect a random distribution of R 's, hence a scatter-diagram of z 's, which is what is actually observed²¹ in the case of quasars (anomalously red-shifted star-like objects), which are now seen to be merely ordinary though compact stars, whose red-shifts are to be considered "anomalous" only if one ignores Mach's Principle. [Cf. Figures 1,2 of Appendix 3C.]

In Appendix 3 it is shown that the relative values of G at perihelion (1/3 AU) and aphelion (1 AU) of the Helios satellite satisfy

$$(34) \quad \frac{G_{\text{per}}}{G_{\text{aph}}} = 1 + \frac{46 \times 10^{-9}}{(|\omega| - 2)} \approx 1 + 0.9 \times 10^{-9}$$

for $\omega \approx -52$, which is compatible* with the experiments analyzed in Appendix 2.

In GR we have

$$(35) \quad G_{\text{per}}/G_{\text{aph}} = 1 ,$$

while in JBD

$$(36) \quad \frac{G_{\text{per}}}{G_{\text{aph}}} = 1 - \frac{46 \times 10^{-9}}{\omega + 2} \approx 1 - 1.3 \times 10^{-9}$$

where $\omega \approx 25$ is the latest value suggested by JBD adherents. The clear-cut distinctions between (34)-(35)-(36) provide a new critical experimental test to discriminate between the present theory, GR, and JBD.

3.2. Asymptotic closed form solution for a homogeneous, isotropic hyperbolic universe: G increases linearly with time.

In Appendix 4 we consider a Friedmann-like hyperbolic cosmology,

$$(37) \quad k = -1 , \quad q < \frac{1}{2}$$

where k has its usual meaning, as do q and H (cf. (20)). The mean density $\rho = \rho(t)$ and cosmic radius $R = R(t)$ as functions of cosmic time t satisfy

$$(38) \quad M = \frac{4}{3} \pi R^3 \rho \equiv \text{constant}.$$

Defining

$$(39) \quad G = \left(\frac{2|\omega| - 4}{2|\omega| - 3} \right) \frac{1}{\phi} , \quad \phi_1 \stackrel{d}{=} \frac{6M}{2|\omega| - 3} ,$$

we can find ϕ and R by solving, simultaneously,

$$(40) \quad \dot{\phi} = - \phi_1 \frac{t}{R^3} ,$$

$$(40b) \quad \frac{\dot{R}}{R} = \frac{1}{2} \left[\left| \frac{\dot{\phi}}{\phi} \right| + \left\{ \left(\frac{2|\omega| - 3}{3} \right) \left[\frac{4}{t} - \left| \frac{\dot{\phi}}{\phi} \right| \right] \left| \frac{\dot{\phi}}{\phi} \right| + \frac{4}{R^2} \right\}^{\frac{1}{2}} \right] > \frac{1}{R} .$$

*For conservatism, this ω is chosen to correspond to $\gamma = 1.02$.

Note that

$$(41a) \quad \frac{\dot{G}}{G} = - \left(\frac{\dot{\phi}}{\phi} \right) = \left(\frac{\dot{\phi}_1}{\phi} \right) \frac{t}{R^3} > 0 ;$$

note also the new result

$$(41b) \quad - \frac{\ddot{R}}{R} = qH^2 = \frac{\dot{G}}{G} \left\{ \frac{|\omega|}{3} \left[\frac{1}{t} - \left(\frac{\dot{G}}{G} \right) \right] + \left(H - \frac{1}{t} \right) \right\} .$$

From these we can prove rigorously that

$$(42) \quad G > \left(\frac{|\omega| - 2}{3} \right) \frac{c^3 t}{M} ,$$

thus rigorously confirming the central idea of Milne's cosmology. To get a rough estimate of (\dot{G}/G) , we shall insert into (41b) Sandage's latest (1974) values $H^{-1} \cong 2 \times 10^{10}$ yrs, $q \cong 0.05$ and assume $H \cong 1/t$; then, setting $\omega \cong -30$, say, (41b) yields

$$(43) \quad 0 \times \frac{\dot{G}}{G} \cong \left(\frac{3}{|\omega|} \right) qH \cong 3 \times 10^{-13} .$$

This is far too small to be detectable by any of the present Earth-Moon laser observations, which can at most detect effects $\cong 10^{-11}$.

3.3. Toward a unified field theory: strong nuclear force's repulsion just an aspect of gravity. (No ad hoc hypothesis of "strong gravity" needed.) Two test-particles near LS radius of a third particle attract infinitely (explanation of quark's triplet nature?) Similarity between principal features of present theory and those of the Einstein-Hlavaty asymmetric, non-metric theory.

These results have already been derived in connection with (31)-(32); with (30); and with (9)-(10) in comparison to (30).

The Hoyle-Narlikar action-at-distance theory²¹ is basically similar to,

and yields results similar to [including Dirac's $(\dot{G}/G) < 0$] the JBD theory; hence its present version must be rejected for the same contra-indications, though presumably a modification is viable.

The present theory can be derived (following Jordan¹⁴) by assuming that the spacetime continuum is really a 5-dimensional "projective" spacetime. A virtually identical theory can be derived from the Einstein-Hlavaty asymmetric, non-metric theory¹² of an affinely connected spacetime, wherein (see refs. ff. (9))

$$(44) \quad G = \frac{G}{1 + \mu(f_{\alpha\beta}f^{\alpha\beta})}, \quad (\text{EINSTEIN-HLAVATY})$$

where $f_{\alpha\beta}$ is the Maxwell electromagnetic tensor, so that, by linearization, variations $\xi = \Delta(f_{\alpha\beta}f^{\alpha\beta})$ in $(f_{\alpha\beta}f^{\alpha\beta}) \propto (H^2 - E^2)$ satisfy

$$(45) \quad \square^2 \xi = 0, \quad (\text{MAXWELL})$$

(where \square^2 is the spacetime D'Alembertian) everywhere except at the point-particle singularities corresponding to elementary charged particles. Similarly, the present Mach's Principle extension of GR involves

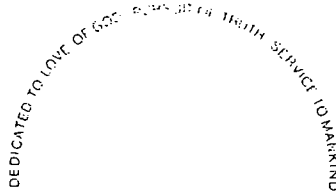
$$(46) \quad G = G/(1 + \xi), \quad (\gamma > 1 \Rightarrow \xi < 0), \quad (\text{MACH-EINSTEIN})$$

where ξ satisfies (45) except at mass-point singularities. Results highly similar to those presented here should therefore be derivable also from the Einstein-Hlavaty theory. Thus Einstein's "final unified field theory" possibly does incorporate Mach's Principle, and possibly does unify the gravitational, strong nuclear and electromagnetic forces (while current work of Weinberg suggests that the weak nuclear forces may be an aspect of electromagnetism). If one accepts the analytic awkwardness of a Dicke-type conformally invariant [variable-mass] version of GR, then a Rainich-Misner-Wheeler type of theory¹³ (cf. Rainich's later version¹⁴) may possibly be usable to display the present theory as a unified theory containing point-particle singularities.

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May 2, 1975

Mrs. Deb Bensen
Gravity Research Foundation
58 Middle Street
Gloucester, MA 01930

Dear Mrs. Bensen:

Enclosed are three copies of an Errata sheet for Appendices 1-4 of my Essay submitted on March 29, 1975. (I have no corrections for the Essay itself.)

Subsequent to the April 1 deadline for the contest, Dr. Van Flandern (APS Bulletin, April) and Drs. Fomalont and Sramek (Physics Today, April) have received considerable national publicity (newspaper articles, etc.) regarding statements which, if taken at face value by the Foundation's judges, would destroy the validity of my Essay. Since the judges can be expected to have read their daily newspapers, it seems only fair to allow me to submit post-deadline rebuttals to their many fallacious statements, which I have done in the new Addendum and Appendix 5, which I trust you will forward to the judges as expeditiously as possible.

Sincerely,

Robert W. Bass
Professor of Physics and Astronomy

db

May 2, 1975

ADDENDUM TO EXHIBIT A

The omitted algebra and calculus details of verification of equations (23) - (50) of Appendix 3A occupy about a hundred handwritten pages. It seems fair to leave that to the reader since Brans and Dicke (1961) did just that in reporting the Misner-Brans solution in isotropic coordinates; see also Brans (1962) where again the details were omitted. It is more difficult to do this work in standard coordinates, but ultimately more useful; and there is no closed-form change of variables which would enable one to pass from isotropic to standard coordinates or conversely. Hence the solution (23) - (50) is an entirely new result. The physical consequences of (23) - (50) are so novel that it is gratifying to report that my claims regarding this have already been checked in part and confirmed by Dr. B. Kent Harrison. Dr. Harrison, author of "Exact three-variable solutions of the field equations of general relativity," Phys. Rev., vol. 116 (1959), p. 1285 [cf. also his papers: ibid., vol. 137 (1965), p. B488, p. B1644; and J. Math. Phys., vol. 9 (1968), p. 1744 and vol. 12 (1971), p. 653] is co-author with Thorne, Wakano and Wheeler of Gravitation Theory and Gravitational Collapse, University of Chicago Press, 1965, and a recognized expert on closed-form field-equation solutions and gravitational collapse. He has subsequent to but independently of my work confirmed that at my generalization (44) of the classical Laplace-Schwarzschild radius, which he refers to as the Bass radius, it is indeed the case that $G \rightarrow +\infty$ as two test-particles or a gravitational clock used to measure G approaches this surface externally. He has also found that the Riemann tensor and the geodesic equations of motion become singular as the Bass radius is approached externally; from this he deduces that the Laplace-Schwarzschild-Bass radius does, in fact, define a topologically spherical surface which is impenetrable by matter particles.

In this connection, it should be noted [cf. Janis, Newman and Winicour (1968), Thorne and Dykla (1971), Hawking (1972), and McIntosh (1974)] that this result, concerning the transformation of the Laplace-Schwarzschild event horizon into a clearly singular surface, was already known in JBD for $\omega > -3/2$ and $\omega \rightarrow +\infty$. However, Hawking's theorem about black holes in JBD for $\omega > -3/2$ being identical to those of GR does not apply to the present theory, wherein $\omega < -2$; in fact, our solution (23) - (50) is static and spherically symmetric and yet G is not constant, which contradicts some consequences of Hawking's hypotheses.

There is a tendency among some of the authors just cited to transform the topologically spherical surface in question into a point at the origin and call it a "naked singularity." However, this introduction of an inadmissible coordinate system violates one of the most fundamental postulates of General Relativity: covariance of the laws of nature under diffeomorphisms (i.e., differentiable homeomorphisms) according to which neither analytically singular nor topologically non-homeomorphic coordinate transformations are physically admissible.

Approximately simultaneously with me, but quite independently, Drs. L. L. Smalley and P. B. Eby have submitted for publication a theory which, though as yet less complete, is (so far as it goes) almost identical to the present theory: $\gamma > 1$ is deduced from the experiments and a theory with $\omega < -2$ is developed. One superiority in the Smalley-Eby treatment is the quotation of the claim of Noerdlinger (1968), of which I was unaware, that a priori $\omega > 0$ for physical reasons analogous to the Landau-Lifschitz proof that $G > 0$; however, Smalley and Eby have discovered a fatal flaw in Noerdlinger's proof. The only difference between their work and mine is that (instead of the observationally required $k = -1$) they have investigated the cosmology of $k = 0$, and [in their analog of my equation

(25) of Appendix 4] they replace my t by $(t - t_c)$ where t_c is an arbitrary unknown integration constant, specifying (if not zero) some future time at which G must stop decreasing and start increasing; thus, they preserve the option of concordance with $\dot{G}/G < 0$ at the present epoch, although ultimately, as $t \rightarrow +\infty$, they agree with my result that $\dot{G}/G > 0$. (For this and other reasons I regard it as preferable to choose $t_c \equiv 0$ from the start.)

I have, perhaps unfortunately, assumed throughout that the reader was already inclined to regard the incorporation of Mach's Principle in GR as at least a desirability if not a necessity; the most persuasive and compelling arguments to this effect which I have seen are those of Bondi (1960) and Sciama (1961); in this connection one must not forget the arguments of Dicke (1961, reprinted 1964) regarding the Lense-Thirring effect, which applies word-for-word to the present theory after the simple substitution $\omega = -|\omega|$.

It should be noted that Das and Narlikar (1975), which appeared after this paper was submitted, have tried to develop within GR a theory of quasars quite similar to that of our Appendix 3C, except that they are forced by Bondi's important theory [$r > (9/8) r_{LS}$] to make the ad hoc hypothesis that deep in the interior of the star there is an electromagnetically transparent shell of stellar matter in some hitherto unconceived physical state.

During April 1975, considerable national publicity (UPI Press Release and a lengthy news item in Physics Today) was given to the forthcoming publication [Ap. J., August 1975] of a paper by Drs. E. Fomalont and R. Sramek alleging to have confirmed GR and conclusively ruled out JBD. (This work was analyzed by us as Experiment Number 24 in our Appendix 1.) Unfortunately they have been quoted or misquoted in the media as having made several over-simplified statements which the present theory demonstrates irrefutably are far from correct. Consider, e.g.: "they then note that for such a high value of the coupling

constant, the scalar-tensor and Einstein theories are not very different in what they say about the early stages of the Universe or near massive objects." A glance at our Appendices 3ABC and 4 shows how fallacious such views actually are. This fallacy is precisely identical, mathematically, to the assumption that for extremely small values of the Reynolds Number there is no essential difference between ideal Eulerian fluid theory and the Navier-Stokes hydrodynamics of Newtonian ["real"] fluids: the volume occupied by boundary layers and vortex sheets or transition surfaces between jets, wakes, cavities, etc., is indeed small, but the physical consequences (e.g., non-zero drag!) are overwhelmingly different. This is because in both cases one has, technically, a singular perturbation problem. Rosen (1970), following Janis, Newman, and Winicour (1968), has already used precisely this idea in his second proof of the non-physical nature of the Schwarzschild interior. Indeed, in such a situation, convergence to the limit is not uniform, as in Gibbs' "jump" Phenomenon with Fourier Series: for very large $|\omega|$, the present theory is indeed indistinguishable from GR except in a thin "boundary-layer" at the Bass radius; the physical unacceptability of such a phenomenon constitutes a rigorous reductio ad absurdum which leaves no alternative to the realization that some stronger form of Bondi's theorem [$r > (9/8) r_{LS}$] must be true even in GR conceived as the limit of the present theory as $\omega \rightarrow -\infty$ [recall ideal fluids as the non-uniform limit of real fluids!], which realization necessitates the final abandonment of the concept of complete gravitational collapse as an untenable proposition.

Acknowledgment. I am most grateful to Drs. B. Kent Harrison, S. Neil Rasband, Robert L. Forward, Larry L. Smalley, and Thomas C. Van Flandern for kindly calling to my attention the following additional references to closely related investigations.

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