A STABILITY-RHOBUSTNESS MARGIN IN DISCRETE TIME

by

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Excerpt from a letter to Prof. Wilson J. Rugh [**1**] of the Dept. of Electrical & Computer Engineering at the Johns Hopkins University dated August 13, 2002 in which I told him about my having generalized from analog time to discrete-time (in connection with an evening course on DSP for the local branch of Florida Inst. of Technology that I was then teaching) my 'Rhobustness' Criterion that I had first published for analog-time systems in 1956 [**2**] and then improved in various subsequent publications [**3**] – [**6**].

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I have just derived what I believe is an elegant generalization from continuous-time systems to discrete-time systems of my theory of "rhobustification" of control and observer systems, as follows:

In your book [**1**] you prove that if the system

$$x^{k+1} = A \cdot x^k + B \cdot u^k, \quad u = -K \cdot x,$$

is stabilized by state-feedback, to become [closed-loop]

$$x^{k+1} = A_{cl} \cdot x^k, \quad x^0 = x^o, \quad A_{cl} \equiv: A - B \cdot K,$$

then there must be positive numbers $\gamma \geq 1$ and $\lambda < 1$ [$\lambda \equiv: \max\{|\mathrm{eig}(A_{cl})|\}$] such that

$$\|x^k\| \leq \gamma \cdot \|x^o\| \cdot \lambda^k, \quad (k = 1,2,3, \ldots).$$

I now define a <u>*stability "rhobustness" margin*</u> ρ, ($0 < \rho < 1$), by $\rho \equiv: (1 - \lambda)/\gamma$, and prove that if in <u>***NONSTATIONARY/NONLINEAR & EXTERNALLY-FORCED***</u> actuality the system is better-modeled by the 'perturbed' system

$$x^{k+1} = A_{cl} \cdot x^k + f(k, x^k) + g(k), \quad (k = 1,2,3, \ldots),$$

where there exist (κ,δ) such that, for all x and k, $\|f(k, x)\| \leq \kappa \cdot \|x\|$, $\|g(k)\| \leq \delta$, and where $\kappa < \rho$, then

$$\|x^k\| \leq \gamma \cdot \|x^o\| \cdot \Lambda^k + \delta/\rho, \quad \Lambda \equiv \{ \lambda + [\kappa/\rho](1 - \lambda)\} < 1, \quad (k = 1,2,3, \ldots).$$

Hence the <u>***larger***</u> is ρ, the <u>***LESS SENSITIVE***</u> is the actual system to *UNMODELLED* **cross-couplings** & other *NONSTATIONARY* and/or *NONLINEAR* effects, as well as *EXTERNAL FORCING!*

References

[**1**] Rugh, Wilson J., *Linear System Theory*, 2nd ed., Prentice Hall, 1996.

[**2**] Bass, Robert W., "Equivalent Linearization, Nonlinear Circuit Synthesis, & the Stabilization & Optimization of Control Systems," *Proceedings,* 1956 Symposium on Nonlinear Circuit Analysis, Brooklyn Polytechnic Institute MRI Symposia Series, vol. VI (1957), *pp*. 163-198.
[**3**] Bass, Robert W., "Robustified LQG Synthesis to Specifications," *Proceedings,* Fifth Army Coordinating Group on Modern Control, Picatinny Arsenal, NJ, 1983, *pp*. B15-B93.
[**4**] Bass, Robert W. & Dean Zes, "Robust Tuning of Kalman Filters," *Proceedings,* American Control Conference (1985), Boston, MA, *pp.* 179-181.
[**5**] Bass, Robert W., "ρ-Synthesis: A 'Rhobustness' Margin for Unstructured Nonlinear and Time-Varying Deviations," *Proceedings,* 30th ***IEEE*** CDC (1991), Brighton, U.K., *pp*. 2531-2537.
[**6**] Bass, Robert W., "On 'A Tutorial Comparison of Three Multivariable Stability Margins'" *Proceedings*, First ***IEEE*** Regional Conference on Aerospace Systems, Westlake Village, CA, 1993, *pp*. 856-862.