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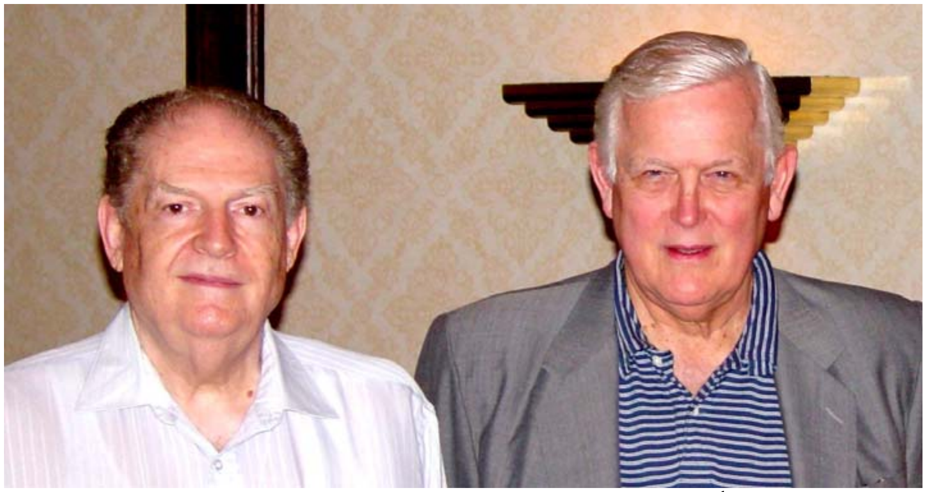
Some Reminiscences of Control & System Theory in the Period 1955-60

The following is the text of Dr. R. W. Bass's talk, presented (3/18/02) at the banquet/speaker event of the 34th Southeastern Symposium on System Theory (SSST), hosted by the UAH/ECE Dept.

***Real Time* would like to express appreciation to Dr. Bass for providing this written text and granting permission to reproduce it here.**



Dr. Robert W. Bass and Dr. Rudolf E. Kalman at the 34th SSST.

It is a singular privilege and honor and also a great pleasure to introduce to this distinguished audience my old friend & colleague, Prof. Rudolf Kalman.

The **Kalman Filter** provides a **critical enabling technology of the Space Age!**

It is well known that by now more than 100,000 papers have been published with the key words "Kalman Filter" in the Title or Abstract, and the last time I checked (about 1995) there were appearing some 200 issued Patents per year from the US Patent Office which carried the same Key Words in either Title or Abstract. More importantly, an official Raytheon statement credits the "information-theoretic architecture of the Kalman Filter" as "mission-critical to the success of the Patriot Missile in shooting down Iraqi SCUDS over both Israel and Saudi Arabia," and others have opined that the essential role of the Kalman Filter in Project Apollo's manned soft lunar landing combined with President Reagan's subsequently credibly-proposed SDI initiative convinced the former USSR to transform itself into a less-threatening nation -- indeed for a decade Russia has been no longer regarded as a potential military adversary, and presently is actually an ally.

Kalman not only discovered what many people have recognized as the most important innovation in systems engineering since World War II but almost single-handedly laid the foundations for the modem mathematical approach to linear systems analysis & synthesis -- an achievement comparable to Euclid's axiomatization of geometry or Newton's formulation of a comprehensive theory of dynamics.

Speaking of achievements I am reminded of what Sir Humphrey Davy replied when asked what he considered to be his life's greatest achievement: without hesitation he answered that his greatest achievement had been that "I discovered Michael Faraday!" Similarly I flatter myself that MY own greatest achievement was that "I discovered Rudolf Kalman!"

In a moment I'll explain what I mean by that claim. However I should preface my remarks by the comment that I have been asked to provide by means of personal reminiscences and anecdotal nostalgia some flavor of the early days when so-called Modem Control & Estimation Theory was being discovered and established.

During the academic year 1955-56 1 was a postdoctoral student of Solomon Lefschetz in the mathematics department at Princeton University. In March 1956 1 attended an on-campus ASME meeting at which I heard a speaker named Rudolf Kalman make a presentation regarding analysis of piecewise-linear systems (such as linear-saturating systems). At that time I fancied that I knew something about this subject because while at Johns Hopkins I had written a 356-page typed ICR Report on *Relay and Discontinuous Systems* which had been accepted for publication by Princeton University Press in book form and in which I had reviewed scores of published papers on the subject. But Kalman's approach struck me as so stunningly original that as I said at the time, "I nearly fell out of my chair!"

I soon learned that Rudolf was an MIT-trained electrical engineer who after designing and building an adaptive control system at DuPont was now pursuing a doctorate at Columbia University.

On that occasion I gave Rudolf a copy of a paper which I had submitted to the following month's Brooklyn Polytechnic Institute *Symposium on Nonlinear Circuit Analysis* in which I employed state-variable techniques that I had learned from Wintner, Hartman & Lewis at JHU and from their and Lefschetz's former student Richard Bellman. In that paper I used Liapunov's Second Method and the Gronwall-Bellman Lemma to define what I called then the "amount ρ of [structural] stability" of a linear system but which in today's terminology would be called a Stability Robustness Margin $\rho = \rho(F)$. Specifically, if $\Phi(t) = \exp(F\cdot t)$ is the system's state- transition matrix, then for all times $t \geq 0$

$$||\Phi(t)|| \leq \gamma\cdot\exp(\lambda\cdot t), \quad \lambda > 0, \quad \gamma \geq 1, \quad \rho = \lambda/\gamma \leq 1/\mu,$$

where μ is the celebrated 1986 robustness criterion of Caltech's famed **Mu-Synthesis** creator John Doyle, who was gracious enough some 35 years later to tell his graduate students in my presence that he had been “amazed” to learn that in 1956 1 had published a lower bound to the reciprocal of his Mu criterion for robust synthesis which had partially anticipated his own important discovery by some three decades.

Accordingly I flatter myself that it was from my 1956 paper that Kalman first appreciated the power of two of the three perspectives which he used in order to discover the by now ubiquitous Kalman Filter, namely the **State Space** approach and **Liapunov's Second Method.** But regarding the third perspective, namely **Wiener Filtering** and related stochastic-process oriented results developed and fostered at MIT, I had known nothing and therefore would never have been able to conceive of, much less even conjecturally formulate, Kalman's epochal discovery. But today, as you will soon hear, Kalman regards the stochasticity aspects as of less significance than mere uncertainty, whether probabilistic or deterministic.

Most systems engineers know that **all** modem aerospace and marine transportation systems, both civilian and military, depend upon the Kalman Filter as a mission-critical component of their guidance, navigation and control (**GNC**) systems. Likewise Kalman Filtering is essential for Fire Control in modem artillery. This state-variable estimation algorithm works online in real-time to estimate the unmeasured state-variables from the over-all system's known dynamics together with feedback of those variables actually instrumented for real-time measurement. Accordingly the Kalman Filter algorithm's embodiment can be likened to a sort of **synthetic super-sensor suite.** In Kalman's own words, the dynamics-based Kalman Filter turned out to be more important than the purely stochastic Wiener Filter because "Newton is more important than Gauss!"

In 1955 Vice President George Trimble of the Martin Co. (later Martin-Marietta and now Lockheed-Martin) sought to establish an industry-sponsored Research Institute for Advanced Studies (RIAS) in hopes of basic-research-derived technical breakthroughs that could assist our **national defense** in what was then called the **Cold War** for containment of expansive communism. Trimble appointed Martin's accomplished electronics manager, Welcome Bender, to recruit researchers, staff and direct RIAS in a Baltimore residential suburb. Bender's first appointment was a recent JHU Ph.D. in physics, Lou Witten, an internationally recognized expert in gravitational physics whose son Edward Witten is today a renowned string theorist at the Institute for Advanced Study in Princeton. Noting that it was commonly acknowledged that the USSR was far ahead of the West in the field of **nonlinear mechanics**, Lou Witten recommended that Bender seek assistance from Solomon Lefschetz at Princeton, who was by then highly active in translating Russian papers in this field into English and in promoting nonlinear mechanics in the USA from his status as a Member of the National Academy of Sciences.

Lefschetz, a Fields Medalist, was already a world-renowned pure mathematician of the highest stature and a Professor Emeritus at Princeton. When Bender and Witten visited Princeton to make an informal presentation of their plans regarding RIAS, in hopes of recruiting Lefschetz to direct an activity in nonlinear mechanics and related fields, such as control theory, I was in the audience and by asking positively-worded questions about their plans made it clear that it sounded good to me though at the time Lefschez himself was understandably reluctant to leave Princeton for Baltimore.

When Lefschetz appeared to be unreceptive to any putative RIAS offer, they made me an offer of employment at RIAS which I accepted in June 1956 though it was a hard decision to select RIAS over a by then formally-awarded NSF Postdoctoral Fellowship to continue at Princeton during 1956-57.

Later I was able to persuade Lefschetz to accept an offer from RIAS to commute by train to Baltimore and direct an activity in nonlinear mechanics whose initial goal was to compete with Soviet activity in the same field. Years later I counted out that of the first 22 theoreticians to whom Lefschetz had made permanent or visiting offers at RIAS, about 11 of them had already been Lefschetz colleagues or protégés, such as Joseph LaSalle and Lamberto Cesari, but the other 11 of them (including Andre & Seibert, Kalman, Hale & Gambill, Pipino, Bucy, and Kushner) had been theorists whom I had first called to Lefsehetz's attention directly or indirectly (e.g. by recommending Kalman who in turn recommended Bucy, etc.).

Kalman's nomination of Bucy was particularly inspired because Bucy soon proved that the well-known Ricatti Equation of the Calculus of Variations was in the case of finite-dimensional systems equivalent to the Wiener-Hopf Equation of stochastic filtering theory, and collaborated fruitfully with Kalman in generalizing all of Kalman’s discrete-time results to the continuous-time case, where one now speaks of the Kalman-Bucy Filter. Also Kalman, Bucy and Englar produced at RIAS, under contract to NASA, the grandfather of all Automatic Synthesis Programs, the famous ASP-C program of 1965, which in FORTRAN could cope with dimensions as high as n = 30. Today the Peacekeeper ICBM is known to employ an onboard real-time Kalman Filter of state-space dimension n exceeding n = 100 !

I well remember that when at Princeton in 1957 I first tried to tell Lefschetz how brilliantly original Kalman was, a world-famed and prize-winning European mathematician turned to me and said: "Tell me: *WHY* are you so interested in this **little** engineer?" [He was using “little” as a synonym for “mere.”]

After I had worked at RIAS for less than a year, and just before Lefschetz did actually hire Kalman, I had to take a two-year leave of absence in order to fulfill my ROTC obligation by active duty service in the Air Force starting in May, 1957. When I returned to RIAS in May, 1959, I became absorbed in my own efforts to find a closed-form analytical expression for the nonlinear state-variable feedback control law of Time-Optimal or "bang- bang" control systems, and therefore was not aware of what Rudolf Kalman had been working on.

One day I said to Kalman quite naively, "If you have n state variables then you need n sensors."

"No, Bob, that's not true," replied Rudolf. "If a system satisfies my criterion of Observability, then you can optimally estimate all unmeasured state variables by using the ones that are measured together with the system's known dynamics. I have been shouting that from the rooftops for the past year! Haven't you been listening?"

Later I asked Rudolf how one could be sure that one actually "knew" the dynamics of the system being controlled. Quoting something else which he had learned at MIT and which I had never heard of, Rudolf replied, "In principle, that's easy! You just take the cross-correlation of the system's output with its command input and then in a suitable sense divide it by the input's auto-correlation in order to get the input-output transfer function!"

During the years just before and after Kalman accepted a professorship at Stanford in 1964 he published algebraic results pertaining to **realization theory,** or modeling of linear input-output systems, which laid the groundwork for a stunning discovery by his graduate student B. L. Ho. I am referring to Ho's doctoral dissertation's main result, published in 1966 as a joint paper with Kalman, which I regard as the most profound theorem pertaining to the System Identification. (ID) problem. Firstly, if noise is negligible, then from input-output measurements one may compute the so-called Markov parameters, or coefficients of a Taylor-series expansion in the complex-frequency domain of an empirical transfer function. Secondly, arrange the Markov parameters into an infinite Hankel matrix, each of whose elements is an ℓ–by-m matrix in the case of ℓ outputs and m inputs. Then the rank n of this matrix defines the minimal dimension of a state-space model of the system! Moreover, by elementary matrix algebra one may compute from the principal $n \times n$ sub-block of the Hankel matrix a triad of matrices (F, G, H) having respectively dimensions $n \times n$, $n \times m$, $\ell \times n$ and which are called the dynamical coefficient matrix, the input coupling matrix (or actuator kinematics matrix) and the output coupling matrix (or sensor kinematics matrix). Furthermore the pair (F,H) satisfies Kalman's criterion of Observability (which enables applicability of the Kalman Filter to estimate optimally all n state variables from the ℓ sensed outputs) and the pair (F,G) satisfies Kahnan's criterion of Controllability (which enables one to derive the optimal Kalman Regulator Law for state-variable feedback control). Furthermore, by Kalman's important Principle of Duality, results in asymptotic estimator theory may be converted into results in control theory, and conversely, by simple matrix transposition operations. By my own Algebraic Separation Theorem, one may design the control system as if all n state-variables were measured and available for feedback, then design a Kalman Filter to estimate them, and combine the two results into an over-all stable system whose $2n$ poles combine those of the ideal regulator and the optimal filter. Moreover it can be proved by Stochastic Optimization Theory, in what some term the Guidance/Navigation Separation Principle, that such a $2n$-pole system is a genuinely optimal system of the so-called LQG type. Here **L** refers to the assumption of linearity employed in the Ho-Kalman Identification Lemma, while **Q** refers to the fact that the Kalman Regulator Law minimizes the integral over future time of a pre-specified arbitrary quadratic form in the state-variables and control variables. Finally, the **G** refers to the fact that the Kalman Filter provides unbiased minimal-variance estimates of the state variables when the process disturbances and measurement noises are all Gaussian white-noise processes.

The only fly in the ointment of LQG theory is that the resultant "optimality" is very fragile if the actual disturbances and noises have covariance intensity matrices different from those assumed during the design. Here I and my collaborator Dean Zes have published a theory of Robust Tuning of a Kalman Filter in which we showed how to choose fictitious covariances that maximize my 1956 Stability Robustness Margin ρ to produce a system optimally insensitive to whatever the off-nominal noise may be. The dual of this enables us to engage in Robust Tuning of a Kalman Regulator by maximizing its closed-loop Robustness Margin $\rho = \lambda/\gamma$. This can be understood as forcing the system to have the <u>relatively</u> fastest response time or largest $\lambda > 0$ possible while simultaneously constraining increases in its overshoot coefficient $\gamma \geq 1$, i.e. by selecting the "most negative" real parts of its closed-loop poles that are compatible with simultaneously minimizing the associated residues. In short, a "rhobustified" system behaves like a scalar system of transfer function $\gamma/(s + \lambda)$ wherein $\rho = \lambda/\gamma$ is maximized.

In *discrete time*, which is normally required for actual implementation of a Kalman Filter or Regulator, one replaces the preceding triad (F,G,H) [in the Linear Time Invariant (LTI) case] by a triad (A,B,C) where A is the state-transition dynamics matrix, B is the input kinematics matrix, and C is the output kinematics matrix, and the exponentially-asymptotic *stability criterion* becomes $\alpha < 1$ where $\alpha = \alpha(A)$ is the largest absolute value of the characteristic roots of A, in which case there exists a real number $\gamma \geq 1$ such that $\|A^k\| \leq \gamma \cdot \alpha^k$, for all positive integers $k = 1, 2, 3, \ldots$. Here my Robustness Margin $\rho = (1 - \alpha)/\gamma$, and, similarly, there is a stochastic-disturbance rejection factor or Fidelity Margin $\sigma = (1 - \alpha^2)^{1/2}/\gamma$, so that in order to render the closed-loop regulator system or state-estimator system maximally insensitive to neglected cross-couplings, parameter-uncertainties, nonlinearities, non-stationarities, and exogenous disturbances, one may "rhobustify" a design by maximizing the arithmetic or geometric mean of ρ & σ.

Another aspect of rendering a system insensitive to uncertainty is that of replacing the usual stochastic noises and disturbances by waveform-based disturbances modeled by *a priori* defined auxiliary linear systems whose states can be identified by a Kalman Filter and thereby provide synthetic disturbance feedforward capability. This Disturbance Accommodating Control (**DAC**) theory of C.D. Johnson is an effective way to accommodate random-like disturbances having a strong component of systematic or semi-deterministic time-behavior, and my collaborator Dan Hill and I have incorporated it in our "grandson-of-ASP-C" RhoSyn/DAC public domain MATLAB Toolkit available on my website www.innoventek.com . [Added 4/6/07: recently, under a DARPA contract, I developed a novel implementation of the above-mentioned Kalman-Ho (Empirical System ID) Algorithm, based upon Leverrier's Algorithm but dispensing with the infinite Hankel Matrix and going directly from Input-Output data to the Markov Parameters and then to the ID given by

(A,B,C), wherein the state-vector dimension n is found "pragmatically" by choice of the best output-prediction given solely the input.]

In 1960 Rudolf and I journeyed to the first IFAC in Moscow and one of my most-cherished memories is that of sitting next to Rudolf while Vice-Premier Kosygin was extolling the virtues of the 1958 Pontriagin Maximum Principle (whose main special case, namely that of the Adjoint System approach to Time-Optimal Control, I had anticipated in my 1956 paper).

In 1961 1 gave a short-course at NASA Langley on "Modem Control Theory." After returning I mentioned to Rudolf that I had proved that "if a system is Controllable according to your criterion of Controllability, then one may compute a control law which places the closed-loop poles in any prespecified stability-constellation." Rudolf responded, "that's very significant, Bob, because it establishes how fundamental my criterion of Controllability is!"

Indeed, every contribution of Kalman to modem control & estimation theory has turned out to be truly fundamental.

In summary, Kalman's seminal contributions include:

The Kalman Filtering Theory [for real-time online minimal-variance state-vector estimation]
The Linear Quadratic Optimal Control Theory [for optimally minimizing future rms errors]
The Control/Filter Duality Principle
The Canonical Decomposition Theorem for Linear Input-Output Systems
The Algebraic Realization Theory
The Ho-Kalman Lemma re System ID via a Minimal Realization
The discovery of a Liapunov Function for the Lure Problem
Numerical analysis enabling an Automatic Synthesis Program (ASP-C)

For these and other contributions to Systems Engineering theory Kalman has received many well-deserved prizes and medals. These include (as well as various European awards too numerous to mention):

A Kyoto Prize in High Technology
An IEEE Centennial Medal
An IEEE Medal of Honor
Membership in the US National Academy of Science
Membership in the US National Academy of Engineering
A Steele Prize of the American Mathematical Society
An Oldenburger Medal of the ASME
A Bellman Heritage Award of the ACC

Accordingly it is now my privilege to introduce to you the true

Father of <u>Modern Systems Engineering Theory</u>, Professor Rudolf Kalman!