Gravitation Essay

APPENDIX 2

PHYSICAL INCOMPATIBILITY OF GENERAL RELATIVITY AND ITS JORDAN-BRANS-DICKE EXTENSION WITH THE OBSERVED HYPERBOLICITY OF THE UNIVERSE

by

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Abstract. The new rigorous result $R = [(1 - 2q)/2q]\, r_{LS}$, where $r_{LS} = 2GM/c^2$, the Laplace-Schwarzschild "black hole" radius of the mass M, and where R denotes the expanding radius of the visible universe containing a total mass M, which holds rigorously in either a Newtonian or a Friedmannian hyperbolic $[q < \frac{1}{2}]$ cosmology, yields a physical absurdity: that although $R(0) = 0$, there is a cosmic time $t_{LS} > 0$ such that $R(t) > r_{LS}$ for $t > t_{LS}$, i.e., matter escapes from a black hole! The dilemma can be avoided if the Cavendish parameter G is allowed to increase linearly with time as in Milne's interpretation of Mach's Principle. This result exposes a basic physical [though not mathematical] inadequacy of both standard General Relativity [G constant] and its Jordan-Brans-Dicke extension [in which G decreases with time].

1. Introduction and Summary.

The Einstein-Friedmann cosmological models with a constant value of the Cavendish parameter G are mathematically well-defined and cannot yield a mathematical inconsistency.

However, in the hyperbolic case (which has been generally disbelieved and somewhat neglected by many relativists from Einstein to Wheeler) these models contradict a reasonable physical requirement, namely that matter cannot travel faster than light-speed c and that matter cannot escape from a Laplace-Schwarzschild black hole.

For heuristic purposes we begin with a Newtonian-Euclidean universe and use Laplace's 1799 definition of a black hole [see Hawking and Ellis (1973), p. 365]: the escape velocity $(2G\,M/r)^{\frac{1}{2}}$, of a mass M at radius r, equals light-speed c, i.e., $r = 2\,GM/c^2$. If the total mass of the universe is M, and its initial total energy is $E = \frac{1}{2}\,(1 - 2\varepsilon)Mc^2 > 0$, where by definition $0 < \varepsilon < 1$, then by the empirical Hubble's Law and observed radial unboundedness there should be [inductively or heuristically] a radius R_∞ at which $\dot{R}_\infty = c$, and beyond which $R = R_\infty + c(t - t_0) \approx ct$; but, by Newtonian conservation of energy, we shall show that $G = \varepsilon(c^2R/M) \approx \varepsilon(c^3t/M)$, which contradicts the assumed constancy of G.

In the case of General Relativity (GR), we first note that it is easy to prove that (because light-speed tends to zero as R tends to the Laplace-Schwarzschild radius r_{LS} from inside a static black hole) neither light nor matter can escape from inside a static black hole, which is a very reasonable postulate -- hereby assumed -- for non-static black holes also.

However, in the observed case [$0 < q \lesssim \frac{1}{2}$, $k = -1$; Sandage (1974), Gunn (1974)] of a hyperbolic Friedmann cosmology we can prove rigorously that (i) the density $\rho(t)$, and radius of the visible universe $R(t)$, satisfy $M = (4/3)\pi R^3\rho \equiv$ constant; and (ii) a new rigorous result $R = [(1 - 2q)/2q]\, r_{LS}$, where q denotes the deceleration parameter $(-\ddot{R}R/\dot{R}^2)$ and where $r_{LS} = 2\ GM/c^2$ denotes the Laplace-Schwarzschild radius of the mass M; while (iii) both $q(t)$ and $1/R(t)$ are monotone decreasing to zero as cosmic time t increases to infinity; in fact, $0 = R(0) \leqq R(t) \to ct$, and $\frac{1}{2} = q(0) \geqq q(t) \to [2 + (c^3t/GM)]^{-1}$, as $t \to +\infty$. In the spirit of the Milne-McCrea Theorem, it should be noted that precisely these results (i) - (iii) also hold rigorously for a Newton-Laplace cosmology, *without* using Hubble's Law explicitly.

Hence in either a Newtonian or a Friedmannian model there is a finite cosmic time t_{LS} at which $q(t_{LS}) = \frac{1}{4}$, which implies that $R(t_{LS}) = r_{LS}$ and that, rigorously, $R(t) \succ r_{LS}$ for $t > t_{LS}$.

Thus we have deduced rigorously the seeming *physical absurdity of matter escaping* from an expanding black hole in a finite interval of cosmic time!

The solution to this dilemma is to incorporate Mach's Principle in GR via a scalar-tensor field theory which allows G to increase linearly with t, as in Milne's cosmology where $G = c^3t/M$.

These results contradict the Jordan(1955)-Brans-Dicke (1961) theory even more strongly than they contradict GR, because in the JBD theory G *decreases* linearly with t.

We turn now to derivation of the above-claimed results.

2. Newton-Laplace Cosmology.

The Milne-McCrea(1934) Theorem shows that the large-scale dynamics of a homogeneous isotropic expanding universe of Newtonian point-masses in a Euclidean space is identical with the corresponding result in Friedmann's relativistic model.

Let $\vec{x}_0^{\,i}$ $(i = 1, 2, \ldots, N)$ denote an equidistant distribution

$$(1) \qquad |\vec{x}^{i}_{o} - \vec{x}^{j}_{o}| = 1 \qquad (i, j = 1, 2, \ldots, N)$$

of a large number N of point-particles in the interior and on the surface of a solid sphere in Euclidean 3-space. Let each point move outward radially in time t by the same proportionality factor R, i.e.,

$$(2) \qquad \vec{x}^{i} = \vec{x}^{i}(t) = Rx^{i}_{o}\ , \ R = R(t),\ (0 \leq t < +\infty),$$

under Newton's Laws of Gravitation and Dynamics

$$(3) \qquad m_i \ddot{\vec{x}}^{i} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{G\, m_i\, m_j}{|\vec{x}^{i} - \vec{x}^{j}|^{3}} (x^{i} - x^{j}),\ (i = 1, 2, \ldots, N)$$

where m_i denotes the mass of the i^{th} particle. Now, using (1)-(2), and dividing (3) by $m_i > 0$, we find, for arbitrary $i \neq j$,

$$(4a) \quad \ddot{R}\, \vec{x}^{i}_{o} = -\left(\frac{G}{R^2}\right) \sum_{\substack{k=1 \\ k \neq j}}^{N} m_k (\vec{x}^{i}_{o} - \vec{x}^{k}_{o}) - \left(\frac{G}{R^2}\right) m_j (\vec{x}^{i}_{o} - \vec{x}^{j}_{o})\ ,$$

$$(4b) \quad \ddot{R}\, \vec{x}^{j}_{o} = -\left(\frac{G}{R^2}\right) \sum_{\substack{k=1 \\ k \neq i}}^{N} m_k (\vec{x}^{j}_{o} - \vec{x}^{k}_{o}) - \left(\frac{G}{R^2}\right) m_i (\vec{x}^{j}_{o} - \vec{x}^{i}_{o})\ .$$

Hence, subtracting (4b) from (4a),

$$(5) \qquad \ddot{R}(\vec{x}^{i}_{o} - \vec{x}^{j}_{o}) = -\frac{GM}{R^2} (\vec{x}^{i}_{o} - \vec{x}^{j}_{o}), \quad (i,k = 1, 2, \ldots, N)$$

where, rigorously, M turns out to be exactly the total mass, i.e.,

$$(6) \qquad M = \sum_{k=1}^{N} m_k\ .$$

Thus, for an incoherent gas ("cloud of dust particles") we have, rigorously,

$$(7) \qquad \ddot{R} = -\frac{GM}{R^2}\ , \quad M = \text{constant}.$$

The usual first integral of (7) is

$$(8) \qquad \tfrac{1}{2}\dot{R}^2 - \frac{GM}{R} \equiv \varepsilon = \text{constant} = \frac{E}{M}\text{, say,}$$

or, multiplying both sides by two,

$$(9) \qquad \dot{R}^2 - \frac{2GM}{R} \equiv \frac{2E}{M}\text{,} \quad E = \text{constant,}$$

where E denotes the system's total energy. If $E = 0$ then

$$(10) \qquad \dot{R} = (2GM/R)^{\frac{1}{2}}\text{,}$$

the so-called escape-velocity: solution of (10) yields

$$(11) \qquad R = [(3^{2/3}/2^{1/3})(GM)^{1/3}]\, t^{2/3}\text{,} \quad 0 \leq t < +\infty.$$

Laplace noted in 1799 that if

$$(12) \qquad R = r_{LS} \overset{d}{=} \frac{2GM}{c^2}$$

then

$$\dot{R} = c\text{ .}$$

If $E > 0$, then, by (9), $(\dot{R})^2 > 0$ for all time t, and so the expansion can never cease (if $\dot{R}(0) > 0$ then actually $\dot{R} > 0$ for all t, since $(\dot{R})^2 > 0$ would be contradicted if $\dot{R}$ reversed sign). Recent observational evidence (Sandage, 1974; Gunn, 1974) concludes that $E > 0$, thereby resolving a long-standing discrepancy between E as computed from the observed density of the universe and E as estimated from kinematic observations. Now Hubble's Law represents an induction from observational evidence that at the present epoch t_o

$$(14) \qquad \dot{R} = H(t_o)R\text{ ,}$$

where $H(t_o)$ = the Hubble parameter $\cong$ 55(km/sec)/Mpc. Since very large values of $\dot{R}$ have been observed (which on a Newtonian-Euclidean interpretation are near $\dot{R} = c$) it is only a slight extrapolation of an observed trend to postulate a radius of the visible universe, at which the right-hand side of (14) is so large that

$$\dot{R}_\infty = c = H(t_o)R_\infty \ . \tag{15}$$

Assuming, as did Laplace, that c is an upper bound on material velocities, we find that for $t > t_o$

$$R = R_\infty + c(t - t_o) \simeq ct \ . \tag{16}$$

Now from (9), if we assume that, by definition of ε,

$$0 < E \overset{d}{=} \tfrac{1}{2}(1 - 2\varepsilon)Mc^2 \ , \ 0 < \varepsilon < \tfrac{1}{2} \ , \tag{17}$$

we have then, from (16)

$$c^2 - \frac{2GM}{R} = (1 - 2\varepsilon)c^2 \tag{18}$$

or

$$G = \varepsilon \frac{c^2R}{M} \simeq \varepsilon \frac{c^3t}{M} \ , \quad (t \to + \infty) \ , \tag{19}$$

which <u>clearly contradicts the hypothesis that</u> G <u>is constant</u>.

A completely rigorous, but also physically absurd analogous result, which does not use Hubble's Law (14) can also be derived from (9) by straightford manipulation. Before presenting this stronger physical anomaly, we shall note that (9) applies precisely also in the Friedmann models of General Relativity (GR).

3. Einstein-Friedmann Cosmology.

Here the basic equations, in units with c = 1, are well known to be

$$M = \frac{4}{3}\pi R^3\rho \equiv \text{constant}, \tag{20}$$

$$\dot{R}^2 + k = \frac{8\pi G}{3} \rho R^2, \tag{21}$$

where k = 0 for parabolic universes, and k = ±1 for elliptic or hyperbolic universes, respectively. Strictly speaking, not (20) but

$$R^3\dot{\rho} + 3R^2 \rho \dot{R} \equiv 0 \equiv (R^3\rho)^\bullet \tag{22}$$

is a consequence of GR; however, it is legitimate to <u>define</u> $3M/4\pi$ as the integration constant of (22). This simple definition of visible rest mass M as a constant is not incompatible with the fact that in hyperbolic space ($k = -1$) the volume of a geodesic sphere of radius R is greater than $(4/3)\pi R^3$, because by the Einstein-Friedmann result (22) the quantity M must be constant. From (22), also

$$(23) \qquad (\rho R^2)^{\bullet} \equiv -\rho R\dot{R} \ .$$

Hence, upon differentiating (21) and using (23), we get (assuming $\dot{R} \not\equiv 0$)

$$(24) \qquad 3\ddot{R} = -4\pi G\rho R \ .$$

Upon combining (21) and (24) we find also that

$$(25) \qquad R\ddot{R} + 2\dot{R}^2 + 2k = 4\pi G\rho R^2 .$$

It should be noted that, upon solving (20) for ρ and inserting the result in (21), we obtain

$$(26) \qquad \dot{R}^2 - \frac{2GM}{R} \equiv -k$$

which is identical with the Newtonian model (9) if we can interpret k as $-2E/M$, i.e., if we can set

$$(27) \qquad k = -\frac{2E}{M} \ .$$

To do this, simply replace R in (20)-(21) by a change-of-scale transformation $R \rightarrow RL$ which replaces (27) by

$$\frac{k}{L^2} = -\frac{2E}{M} \ , \ L \overset{d}{=} \left(\frac{M}{2|E|}\right)^{\frac{1}{2}} \ ,$$

or, equivalently, by

$$(29) \qquad k = -\mathrm{sgn}[E] = \begin{Bmatrix} -1, & E > 0 \\ 0, & E = 0 \\ +1, & E < 0 \end{Bmatrix} \ ,$$

and otherwise leaves (21-(25) unchanged.

Accordingly, we shall adopt as our fundamental equations (20), (21), (24), (25), and (29), which then apply equally well to either a Newton-Laplace model or an Einstein-Friedmann model.

To facilitate comparison with experimental observations, we now take

(30) $$k = -1 \qquad \text{(Sandage, 1974; Gunn, 1974)}$$

and define partly dimensionless parameters H, the Hubble parameter, and q, the deceleration parameter, as usual by

(31) $$H = \dot{R}/R \ , \quad q \overset{d}{=} (-\ddot{R}/R)/H^2 \ .$$

Then, by definition,

(32) $$\ddot{R}/R = -qH^2,$$

and we can re-write (21), (24), (25) as

(33) $$\frac{8\pi G\rho}{3} = H^2 + \frac{k}{R^2} = -2\,\frac{\ddot{R}}{R} = 2qH^2 =$$

$$= \frac{2}{3}\left(\frac{\ddot{R}}{R} + 2\,\frac{\dot{R}^2}{R^2} + 2\,\frac{k}{R^2}\right) =$$

$$= \frac{2}{3}\left(-qH^2 + 2H^2 + 2\,\frac{k}{R^2}\right) ,$$

whence

(34) $$\frac{k}{R^2} = (2q - 1)H^2 \ ,$$

and, by (20) and (30),

(35) $$G = \frac{3}{8\pi\rho}\,\{2qH^2\} = \frac{3}{8\pi\rho}\,\{2q\}\,\frac{1}{(2q - 1)}\,\frac{k}{R^2} =$$

$$= \frac{3}{4\pi\rho}\left\{\frac{q(-k)}{(1 - 2q)R}\right\} =$$

$$= \left(\frac{q}{1 - 2q}\right)(-k)\,\frac{R}{\frac{4\pi R^3\rho}{3}} =$$

$$= (-k)\left(\frac{q}{1 - 2q}\right)\frac{R}{M} =$$

$$= \left(\frac{q}{1 - 2q}\right)\frac{R}{M} ,$$

or, in other words (recalling that $c = 1$)

$$\boxed{R = R(t) = [(1 - 2q)/2q]\, r_{LS} , \quad r_{LS} \overset{d}{=} \frac{2GM}{c^2} ,} \tag{36}$$

an apparently hitherto unnoticed result. In the theory of Friedmann models, one can integrate (26), (30) to obtain the fact that both $1/R(t)$ and $q(t)$ are monotone decreasing as t increases, and that, in fact,

$$0 = R(0) \leqq R(t) \to ct \quad \text{as } t \to +\infty. \tag{37a}$$

$$\tfrac{1}{2} = q(0) \geqq q(t) \to [2 + (c^3t/MG)]^{-1} \to 0 \text{ as } t \to +\infty. \tag{37b}$$

Hence we have the *rigorous* result that there must exist a time $t = t_{LS}$ at which

$$q(t_{LS}) = 1/4, \tag{38}$$

whence, from (36),

$$R(t_{LS}) = r_{LS} \tag{39}$$

or, as a direct consequence of $\dot{R} > 0$ for all t, i.e., of the *observational* result (30), we must have

$$R(t) > r_{LS} \quad \text{for} \quad t > t_{LS} . \tag{40}$$

The result (40) is mathematically acceptable but it is physically absurd: even Laplace would have rejected it!

Fortunately, the incorporation of Mach's Principle in GR via a modified Jordan-Brans-Dicke type of scalar-tensor gravitational field theory obviates this absurdity if the new theory if so chosen that Milne's postulate

$$G \approx \frac{c^3t}{M} \tag{41}$$

holds asymptotically for large times t. The physical viability of such a theory will be demonstrated elsewhere (it agrees with all 24 known solar system metric gravitational theory experiments to two standard deviations, i.e., there is only a 5% probability that either GR or JBD is consistent with these experiments), as will be the rigorous derivation of (41) from a covariant metric theory based on a Lagrangian variational principle.

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