APPENDIX 1

GENERAL RELATIVITY AND ITS JORDAN-BRANS-DICKE EXTENSION CONTRADICTED TO TWO STANDARD DEVIATIONS BY THE TWENTY-FOUR KNOWN SOLAR SYSTEM EXPERIMENTS

by

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Abstract. From the 24 known solar system experiments the mean value $\bar{\gamma}$, and its standard deviation $\bar{\sigma}$, for the Eddington-Robertson (PPN) parameters γ, are given by $\bar{\gamma} \pm \bar{\sigma} = 1.10 \pm 0.05$, i.e., if the observation errors are random and normal, then $\gamma > 1$ with about 95% probability. (Acceptance and use of the various experimenters' estimated standard errors and substitution of a correspondingly weighted optimal [or maximum likelihood] estimation process weakens this result to $\hat{\gamma} \pm \hat{\sigma} = 1.02 \pm 0.06$, i.e., $\gamma > 1$ with at least about 60% probability.) Hence it is likely that neither General Relativity [$\gamma = 1$] nor the Jordan-Brans-Dicke extension of it [$-3/2 < \omega < +\infty$, $\{(\omega + 1)/(\omega + 2)\} = \gamma < 1$] is a physically viable theory. A modified scalar-tensor theory which requires $\gamma > 1$ will be presented elsewhere.

Gravitation Essay

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ERRATA AND CORRIGENDA

1. Exhibit A

   a. Page 2, bottom line: strike out first factor 2.

   b. Page 8, line 8: insert [Our Maximum Likelihood estimate is $\gamma = 1.02 \pm 0.01$.]

   c. Page 14, next to bottom line: version[22].

2. Appendix 1

   a. Abstract, line 7: replace "weakens" by "changes" and $\pm$ 0.06 by $\pm$ 0.01; in next line replace 60% by 95%.

   b. Page 3: delete factors (1/N) from (6) and (7),

   c. Page 4, equation (8): replace $\pm 0.20 > 1.01$ by $\pm 0.06 > 1.15$; in (10): replace $\pm 0.088 > 0.92$ by $\pm 0.062 > 0.95$.

   d. Page 5, equation (11): replace $\pm 0.059 > 0.96$ by $\pm 0.018 > 1.004$; equation (12): replace $\pm 0.029 > 0.98$ by $\pm 0.011 > 0.99$; equation (13): replace $\pm 0.06 > 0.96$ by $\pm 0.01 > 1.01$; two lines below (13) replace 60% by 95%.

   e. Page 8: replace $\pm 0.20 > 1.01$ by $\pm 0.06 > 1.15$.

   f. Page 10: replace $\pm 0.088 > 0.92$ by $\pm 0.062 > 0.95$.

   g. Page 11: replace $\pm 0.059 > 0.96$ by $\pm 0.018 > 1.004$.

   h. Page 12: replace last column by 0.06, 0.015, 0.062, 0.018 and replace $\pm 0.029 > 0.98$ by $\pm 0.011 > 0.99$.

   i. Page 14: replace $\pm$ 0.055 by $\pm 0.011 > 1.006$.

3. Appendix 3A

   a. Page 2, equation (7): insert minus sign before right-hand side.

4. Appendix 3B

   a. Insert reference 11:

      A. Einstein, "Stationary systems with spherical symmetry consisting of many gravitating masses," Annals of Mathematics, Ser. 2, vol. 40 (1939), pp. 922-936.

"One of the key goals of the 1970's is to push experimental tests of gravitation theories to their limit, so that, finally, only one theory will remain," according to Will (1972). A thorough analysis (Will, 1973) of virtually all known theories shows that the most viable are Einstein's General Relativity, the Jordan-Brans-Dicke-Bergmann-Wagoner scalar-tensor theories, and the vector-tensor theory. We shall here accept the authoritative opinion of Will (1974) that "For a variety of reasons the Brans-Dicke theory is considered to be the strongest alternative to general relativity." [Actually, the results to be presented below will rule out the vector-tensor theory also, since in that theory $\gamma = 1$.] Now according to Weinberg (1972), "In order to decide whether Einstein, or Brans and Dicke, or someone else, has the right field equations, what must be done is to measure [the Eddington-Robertson or PPN parameters] α, β, and γ." Thus we consider only two cases:

(1) Einstein $\qquad \alpha = \beta = \gamma = 1$

(2a) Brans-Dicke $\qquad \alpha = \beta = 1, \gamma = \frac{\omega + 1}{\omega + 2} < 1,$

(2b) $\qquad -3/2 < \omega < +\infty ,$

where the restriction (2b) is basic to the theory of Brans-Dicke (1961).

To date four classes of experimental observations involving solar system astrophysics have been proposed, and twenty-four such observations have been or will shortly be published. These observations provide error-corrupted parameter measurements

(3) $\qquad \gamma_i \pm \sigma_i \qquad (i = 1, 2, 3, . . ., N), N = 24,$

where σ_i is the i^{th} experimenters' estimated standard error. If we decide not to rely too much on the estimate $\{\sigma_i\}$, we may analyze the measurements

$\{\gamma_i\}$ by finding the mean value $\overline{\gamma}$ and its standard deviation, the standard error $\overline{\sigma}$, from the well known algorithms

(4) $$\overline{\gamma} = \frac{1}{N} \sum_{i=1}^{N} \gamma_i \ ,$$

(5) $$\overline{\sigma} = \frac{\sigma}{\sqrt{N}} \ , \ \sigma = \{\frac{1}{N-1} \sum_{i=1}^{N} (\gamma_i - \overline{\gamma})^2\}^{\frac{1}{2}} \ .$$

(A chi-square test may then be used to decide if the data $\{\gamma_i\}$ approximately fit a normal distribution of mean $\overline{\gamma}$ and variance σ^2.) Alternatively, if credence is given to the estimates $\{\sigma_i\}$, then (cf., e.g., Bevington, 1969) a maximum likelihood, minimal variance, or <u>optimal</u> estimate $\hat{\gamma}$ may be obtained from the algorithms

(6) $$\hat{\sigma} = \{\frac{1}{N} \sum_{i=1}^{N} \frac{1}{(\sigma_i)^2}\}^{-\frac{1}{2}} \ ,$$

(7) $$\hat{\gamma} = (\hat{\sigma})^2 \{\frac{1}{N} \sum_{i=1}^{N} \frac{\gamma_i}{(\sigma_i)^2}\} \ .$$

The results (6) - (7) provide an estimate $\hat{\gamma} \pm \hat{\sigma}$ which is superior to the $\overline{\gamma} \pm \overline{\sigma}$ of (4) - (5) only when the estimates $\{\sigma_i\}$ are valid and meaningful, and so should be treated with some caution, in that, if a particular experimenter were over-optimistic about the precision of his results, then his data would be weighted unduly in arriving at the supposedly "optimal" estimate $\hat{\gamma} \pm \hat{\sigma}$; see Bass (1966). However, it will be seen that, no matter how we process the data, the results announced in the abstract are unavoidable.

The first class of experiment, proposed by Einstein in 1916, has been performed eleven times during and since the eclipse of 1919. Table 1 summarizes the results. [Table 1 is taken, with trivial modifications to achieve uniformity of format, from Weinberg's Table 8, p. 193; cf.his equation (8.5.10), p. 190.] The results are

(8)(I) $\bar{\gamma} = 1.19 \pm 0.11 > 1.08$, $\hat{\gamma} = 1.21 \pm 0.10 > 1.01$.

The second class of experiment concerns what had been known as an anomaly for a half-century prior to Einstein's theory: the residual precession of the perehelion of Mercury by 43.11 ± 0.45 seconds of arc per century [according to the reanalysis by Clemence (1943, 1947) of accurate observations of Mercury since 1765, in which he essentially confirmed Simon Newcomb's 1882 value; for references, see Weinberg, p. 198; also, cf. Weinberg's equation (8.6.10), p. 197]. Thus, as in Table 2,

(9)(II) $\bar{\gamma} = 1.003 \pm 0.000 > 1.00$, $\hat{\gamma} = 1.003 \pm 0.015 > 0.99$.

The third class of experiment, concerning radar-echo time delay, was suggested only recently by Shapiro (1964), and discussed by Weinberg [p. 203-205] in terms of the measurements of Shapiro (1971), which involved passive reflection of radar from Mercury and Venus. An alternative version of this experiment, using the spacecraft Mariners VI and VII as active retransmitters of the radar signals, was performed by Anderson, et al. (1971) and is compared with the passive-mode results in Table 3. Here we have

(10)(III) $\bar{\gamma} = 1.015 \pm 0.015 > 1.00$, $\hat{\gamma} = 1.012 \pm 0.088 > 0.92$.

The fourth class of experiment is conceptually similar to the first, but quite different in technique, and in principle is capable of much greater precision. The ten known experiments of this type are summarized in Table 4. Thus we have

(11)(IV) $\bar{\gamma} = 1.027 \pm 0.025 > 1.00$, $\hat{\gamma} = 1.022 \pm 0.059 > 0.96$.

It can be seen (Table 5) that each of the four classes of experiments, taken separately yields $\bar{\gamma} \pm \bar{\sigma} > 1$.

Now if we wish to weight the four classes equally, we may process the data in Table 5 to obtain

(12) $\bar{\gamma} = 1.058 \pm 0.044 > 1.01$, $\hat{\gamma} = 1.005 \pm 0.029 > 0.98$.

Alternatively, as in Tables 6-7, we may wish to treat all 24 measurements $\{\gamma_i\}$ on an equal basis, in which case we obtain

(13) $\bar{\gamma} = 1.10 \pm 0.05 > 1.05$, $\hat{\gamma} = 1.02 \pm 0.06 > 0.96$.

In conclusion, no matter how we process the measured observations $\{\gamma_i\}$, there is at least a 60% probability that $\gamma > 1$. The conclusion that $\gamma > 1$ renders non-viable the only two theories presently regarded as viable, namely (1) - (2).

Elsewhere I shall present a new scalar-tensor theory, which, unlike the Jordan-Brans-Dicke theory, incorporates Mach's Principle in the precise manner specified as a *desideratum* by Einstein (1922), and which, intrinsically requiring $\gamma > 1$, is perfectly compatible with the results of all 24 known experiments as analyzed here.

Acknowledgements

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Experiment	Date	Site	# of Stars	θ	σ_e	$\frac{1+\gamma}{2} = \frac{\theta}{1.75}$	γ_i	σ_i
1	1919	Sobral	7	1.98"	0.16"	1.131 ± 0.091	1.263	0.183
2	1919	Principe	5	1.61	0.40	0.920 ± 0.229	0.840	0.457
3	1922	Australia	11-14	1.77	0.40	1.011 ± 0.229	1.023	0.457
4	1922	Australia	18	1.84	0.37	1.051 ± 0.211	1.103	0.423
5	1922	Australia	62-85	1.72	0.15	0.983 ± 0.086	0.966	0.171
6	1922	Australia	145	1.82	0.20	1.040 ± 0.114	1.080	0.229
7	1929	Sumatra	17-18	2.24	0.10	1.280 ± 0.057	1.560	0.114
8	1936	USSR	16-29	2.73	0.31	1.560 ± 0.177	2.120	0.354
9	1936	Japan	8	1.705	0.425	0.974 ± 0.243	0.949	0.486
10	1947	Brazil	51	2.01	0.27	1.149 ± 0.154	1.297	0.309
11	1952	Sudan	9-11	1.70	0.10	0.971 ± 0.057	0.943	0.114

$$\overline{\gamma} \pm \overline{\sigma} = 1.19 \pm 0.11 > 1.08$$

$$\hat{\gamma} \pm \hat{\sigma} = 1.21 \pm 0.20 > 1.01$$

(I) Deflection of Light Waves
Table 1

Experiment	Experimenters	$\Delta\phi$	σ_e	$\frac{1 + 2\gamma}{3} = \frac{\Delta\phi}{43.03''}$	γ_i	σ_i
12	Newcomb (1882) Clemence (1943, 1947)	43.11"	0.45"	1.002 ± 0.010	1.003	0.015

$$\bar{\gamma} \pm \bar{\sigma} = 1.003 \pm 0.000 > 1.00$$

$$\hat{\gamma} \pm \hat{\sigma} = 1.003 \pm 0.015 > 0.99$$

(II) Precession of Perehelion of Mercury

Table 2

Experiment	Experimenters	Date	Mode	$\frac{1+\gamma}{2}$	γ_i	σ_i
13	Shapiro, _et al_.	1971	Passive	1.015 ± 0.05	1.03	0.10
14	Anderson, _et al_.	1971	Active	1.00 ± 0.04	1.00	0.08

$$\bar{\gamma} \pm \bar{\sigma} = 1.015 \pm 0.015 > 1.00$$

$$\hat{\gamma} \pm \hat{\sigma} = 1.012 \pm 0.088 > 0.92$$

(III) Radar Echo Time Delay

Table 3

Experiment	Date	Site	$\frac{1+\gamma}{2}$	γ_i	σ_i
15	1969	Goldstone	1.04 ± 0.15	1.08	0.30
16	1969	Owens Valley	1.01 ± 0.12	1.02	0.24
17	1970	Cambridge	1.07 ± 0.17	1.14	0.34
18	1970	Haystack-Goldstone	1.03 ± 0.20	1.06	0.40
19	1971	N.R.A.O.	0.94 ± 0.06	0.88	0.12
20	1972	Westerbork	0.96 ± 0.05	0.92	0.10
21	1972	Cambridge	1.04 ± 0.08	1.08	0.16
22	1972	Haystack - N.R.A.O.	0.99 ± 0.03	0.98	0.06
23	1973	Westerbork	1.038 ± 0.033	1.076	0.066
24	1974	N.R.A.O.	1.015 ± 0.011	1.030	0.022

$$\overline{\gamma} \pm \overline{\sigma} = 1.027 \pm 0.025 > 1.00$$

$$\hat{\gamma} \pm \hat{\sigma} = 1.022 \pm 0.059 > 0.96$$

(IV) Deflection of Radio Microwaves

Table 4

Class of Experiment	$\overline{\gamma} \pm \overline{\sigma}$	$\hat{\gamma}_i$	$\hat{\sigma}_i$
I Deflection of Light Waves	1.19 ± 0.11 > 1.08	1.21	0.20
II Precession of Perehelion	1.003 ± 0.000 > 1.00	1.003	0.015
III Radar Echo Time Delay	1.015 ± 0.015 > 1.00	1.012	0.088
IV Deflection of Radio Microwaves	1.027 ± 0.025 > 1.00	1.022	0.059

$$\overline{\overline{\gamma}} \pm \overline{\overline{\sigma}} = 1.058 \pm 0.044 > 1.01$$

$$\overline{\hat{\gamma}} \pm \overline{\hat{\sigma}} = 1.062 \pm 0.050 > 1.01$$

$$\hat{\hat{\gamma}} \pm \hat{\hat{\sigma}} = 1.005 \pm 0.029 > 0.98$$

Results by Class

Table 5

Experiment	γ_i	Experiment	γ_i
1	1.263	14	1.00
2	0.840	15	1.08
3	1.023	16	1.02
4	1.103	17	1.14
5	0.966	18	1.06
6	1.080	19	0.88
7	1.560	20	0.92
8	2.120	21	1.08
9	0.949	22	0.98
10	1.297	23	1.076
11	0.943	24	1.030
12	1.003		
13	1.03	Mean	1.102

$$\overline{\gamma} \pm \overline{\sigma} = 1.10 \pm 0.05 > 1.05$$

Mean of All 24 Experiments

Table 6

Experiment	γ_i	σ_i	$1/(\sigma_i)^2$	$\gamma_i/(\sigma_i)^2$
1	1.263	0.1829	29.893	37.755
2	0.840	0.4571	4.786	4.020
3	1.023	0.4571	4.786	4.896
4	1.103	0.4229	5.591	6.167
5	0.966	0.1714	34.039	32.882
6	1.080	0.2286	19.135	20.667
7	1.560	0.1143	76.543	119.408
8	2.120	0.3543	7.966	16.889
9	0.949	0.4857	4.239	4.023
10	1.297	0.3086	10.500	13.619
11	0.943	0.1143	76.543	72.180
12	1.003	0.015	4444.444	4457.778
13	1.03	0.10	100.000	103.000
14	1.00	0.08	156.250	156.250
15	1.08	0.30	11.111	12.000
16	1.02	0.24	17.361	17.708
17	1.14	0.34	8.651	9.862
18	1.06	0.40	6.250	6.625
19	0.88	0.12	69.444	61.111
20	0.92	0.10	100.000	92.000
21	1.08	0.16	39.063	42.188
22	0.98	0.06	277.778	272.222
23	1.076	0.066	229.568	247.016
24	1.030	0.022	2066.116	2128.099
		Mean	325.003	330.712

$\hat{\gamma} \pm \hat{\sigma} = 1.017 \pm 0.055$

Maximum Likelihood Mean of All 24 Experiments

Table 7